

NANO 703/703L  
Homework 2  
Due: M-10/3, 10:00 AM

1) Briefly describe the following terms:

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|--------------------------|--------------------------|
| a) bremsstrahlung X-rays | b) characteristic X-rays |
| c) Auger electrons       | d) secondary electrons   |

2) An electron source has brightness  $\beta = 3.6 \times 10^8 \text{ A}/(\text{cm}^2 \cdot \text{sr})$ .

a) An image of the source at point 1 has diameter  $d_1 = 0.62 \mu\text{m}$  and semi-angle of convergence  $\alpha_1 = 8.2 \text{ mrad}$ . Find the probe current  $i_1$

b) A second image of the source at point 2 has diameter  $d_2 = 29 \text{ nm}$  and current  $i_2 = 8.5 \mu\text{A}$ . Find the semi-angle of convergence  $\alpha_2$  (in mrad) at point 2.

3) An electron probe with lateral diameter  $d = 72 \text{ nm}$  diverges at a semi-angle  $\alpha = 18.0 \text{ mrad}$  at a distance  $p = 0.96 \text{ cm}$  in front of a lens with focal length  $f = 0.68 \text{ cm}$ . In back of the lens, find:

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|--|---|
| a) The image distance $q$ ,  | b) the lateral magnification $M$ ,        |
| c) the lateral size $d'$ of the image,                             | d) the angular magnification $M_\theta$ ; |
| e) The convergence semi-angle $\alpha'$ of the image of the probe; |   |

4) Find the net force on charge  $q = -2.0 \text{ nC}$  with velocity  $\mathbf{v}$  and magnetic field  $\mathbf{B}$  given by:

- a)  $\mathbf{v} = -(0.66c)\hat{\mathbf{z}}$ ,  $\mathbf{B} = (0.80 \text{ T})\hat{\mathbf{p}} + (5.8 \text{ T})\hat{\mathbf{z}}$ ;  
 b)  $\mathbf{v} = (0.0089c)\hat{\mathbf{p}} - (0.66c)\hat{\mathbf{z}}$ ,  $\mathbf{B} = (5.8 \text{ T})\hat{\mathbf{z}}$

(Note that  $\hat{\mathbf{p}} \times \hat{\mathbf{p}} = \hat{\mathbf{z}}$ . Also  $1 \text{ T} = 1 \frac{\text{N}}{\text{A} \cdot \text{m}}$ .) Your answers should be vectors with units of N.

5) Consider an objective lens with spherical aberration coefficient  $C_s = 1.8 \text{ mm}$  and a semi-angle of collection  $\beta = 6.0 \text{ mrad}$  acting on electrons with energy  $200 \text{ KeV}$ .

- a) What is the diffraction limit  $\delta_d$  on the resolution?  
 b) What is the spherical-aberration limit  $\delta_s$  on the resolution?  
 c) What is the *combined* resolution limit  $\delta_{\text{net}}$ , including both diffraction and spherical aberration?  
 d) What is the *optimal* semi-angle of collection  $\beta_{\text{opt}}$  of the lens?  
 e) What is the *practical* resolution  $\delta_{\text{min}}$  of the lens?

6) Find expressions for the intensities  $I(\mathbf{r}) = |\psi(\mathbf{r})|^2$  of the wave functions below.

( $\mathbf{k}$ ,  $\mathbf{K}$ , and  $\mathbf{r}$  are real):

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|---|--|
| a) $\psi(\mathbf{r}) = Ae^{2\pi i \mathbf{k} \cdot \mathbf{r} + i\phi}$ , ( $A$ , $\phi$ real)      | b) $\psi(\mathbf{r}) = i \cdot \cos(\theta) \cdot \frac{e^{2\pi i k r}}{r}$ , [ $\theta$ is real]  |
| c) $\psi(\mathbf{r}) = e^{2\pi i \mathbf{k} \cdot \mathbf{r}} e^{-\mathbf{K} \cdot \mathbf{r}/2}$ , | d) $\psi(\mathbf{r}) = \frac{2}{5} \cdot e^{2\pi i \mathbf{k} \cdot \mathbf{r}} + i \cdot \frac{\sqrt{21}}{5} \cdot e^{-2\pi i \mathbf{k} \cdot \mathbf{r}}$ |