NANO 703/703L

Homework 5

Due: M-12/5, 10:00 AM

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1) A CBED pattern acquired at 200 KeV shows a FOLZ ring ($n = 1$) with diameter $2G_1 = 46.4\ \text{nm}^{-1}$. Find:

a) The zone separation *H*;

b) The diameter $2G_2$ of the SOLZ ($n = 2$).

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2) Determine the Miller indices *hkl* for *all* of the lowest-order (smallest non-zero g_{hkl}) reflection(s), including the correct index signs, for an *fcc* crystal in the following zones:

a) $\left[10\bar{1}\right]$ ZOLZ, b) $[22\bar{1}]$ ZOLZ, c) [120] FOLZ, d) $\left[1\bar{2}1\right]$ SOLZ

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3) In a strong two-beam condition, the Bloch waves in a crystal have eigenfunctions:

$$\left|\psi^{(1)}\right\rangle = \begin{pmatrix} C_0^{(1)} \\ C_g^{(1)} \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}, \quad \left|\psi^{(2)}\right\rangle = \begin{pmatrix} C_0^{(2)} \\ C_g^{(2)} \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

The eigenvalues are $\gamma^{(1)} = -\dfrac{1}{2\xi_g}$ and $\gamma^{(2)} = \dfrac{1}{2\xi_g}$

Assume the operator for the imaginary part of the potential is:

$$\hat{A}' = \frac{\alpha}{2}\cdot\begin{pmatrix} \dfrac{1}{\xi_0} & \dfrac{1}{\xi_g} \\ \dfrac{1}{\xi_g} & \dfrac{1}{\xi_0} \end{pmatrix}$$

where α is a small, dimensionless coefficient.

a) Find expressions for the imaginary components of the eigenvalues $\gamma'^{(1)}$ and $\gamma'^{(2)}$ using:

$$\gamma'^{(1,2)} = \left\langle \psi^{(1,2)} \middle| \hat{A}' \middle| \psi^{(1,2)} \right\rangle$$

b) Evaluate $\gamma^{(1,2)}$ and $\gamma'^{(1,2)}$, assuming $\xi_0 = 48\ \text{nm}$, $\xi_g = 96\ \text{nm}$, and $\alpha = 0.10$.

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4) A two-beam dark-field image of an antiphase boundary using 200 KeV electrons shows a reversal from dark/light ($w = 0$) to light/dark ($w = \sqrt{3}$) contrast when the sample is tilted by 8.6 mrad from the Bragg condition.

For the reflection used ($g = 7.3\ \text{nm}^{-1}$), determine:

a) the extinction distance $\xi_{\mathbf{g}}$;

b) the magnitude of the Fourier coefficient $U_{\mathbf{g}}$ of the structure function.

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5) An FFT from a high-resolution image of an amorphous material taken at 200 KeV with an objective lens spherical aberration coefficient of $C_s = 1.940\ \text{mm}$ has an intensity minimum at $u_n = 2.63\ \text{nm}^{-1}$. The next minimum occurs at $u_{n+1} = 3.04\ \text{nm}^{-1}$. Find the defocus Δf of the objective lens (in nm).

6) The object wave function below a sample is:

$$F(x) = 1 + iB \cdot \cos(2\pi g x).$$

The microscope transfer function in reciprocal space is given by $H(u) = A(u) \cdot e^{-i\chi(u)}$.

Find the image wave function $G(x)$ in direct space for the following cases:

a) $A(u) = \begin{cases} 1, & |u| \le g/2 \\ 0, & g/2 < |u| \end{cases}$, $\chi(u) = \begin{cases} 0, & |u| \le g/2 \\ \pi/2, & g/2 < |u| \end{cases}$

b) $A(u) = \begin{cases} 1, & |u| \le 3g/2 \\ 0, & 3g/2 < |u| \end{cases}$, $\chi(u) = \begin{cases} 0, & |u| \le g/2 \\ \pi/2, & g/2 < |u| \end{cases}$

c) $A(u) = \begin{cases} 1, & |u - g/2| \le g \\ 0, & g < |u - g/2| \end{cases}$, $\chi(u) = \begin{cases} 0, & |u| \le g/2 \\ \pi, & g/2 < |u| \end{cases}$