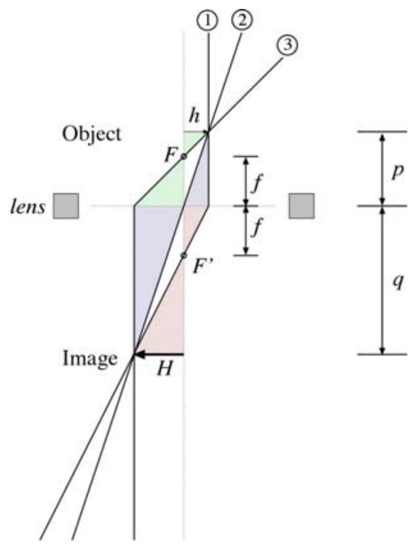
Chapter 5-Magnification and Electron Sources

Lens equation

Let's first consider the properties of an ideal lens. We want rays diverging from a point on an object in front of the lens to converge to a corresponding point on an image in back of the lens. Let's assume that rays parallel to the optic axis passing from the front of the lens through the lens plane converge to a focal point (F') on the back of the lens, and that rays passing through the center of the lens continue undeflected on the opposite side of the lens. The direction of the rays is reversible, so rays parallel to the optic axis on the back of the lens must also pass through a focal point (F) on the front of the lens.



Drawing the lens with an example of each of these rays passing through a point on an object shows that pairs of similar, right triangles are formed by the various intersections. Similar triangles are scaled versions of each other, so the ratios of the corresponding side lengths must be equal. Measuring from the lens plane, we call the object distance p and the image distance q. This gives three equal ratios:

$$\frac{h}{f}=\frac{H}{q-f},\ \frac{h}{p}=\frac{H}{q},\ \frac{h}{p-f}=\frac{H}{f}$$

The focal length on either side of the lens is f. H and h are the distances of corresponding image and object points from the optic axis. Combining any two of these of these ratios results in the ideal lens equation:

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$

Lateral magnification (II)

Lateral magnification M (often just called magnification) is the ratio of the size of the image, measured perpendicular (lateral) to the optical axis, divided by the size of the object.

$$M=\frac{H}{h}$$

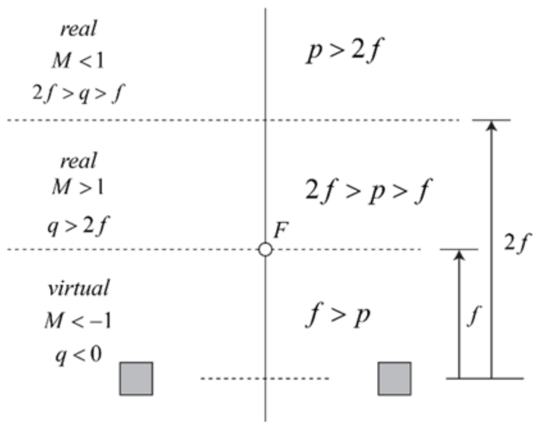
We defined M such that the image is inverted with respect to the object. We can use the ratios above to write M in various ways. It is especially useful to notice

$$M = \frac{q}{p}$$

A magnified ($|M|>1$) image always appears farther from the lens plane than the object, and a demagnified image ($|M|<1$) is always closer. For a fixed f, the position and orientation of the image change depending on where we place the object. Notice

$$q = \frac{1}{\frac{1}{f} - \frac{1}{p}}$$

We had assumed a real image is formed with $q>0$, behind the lens. But some configurations give a virtual image, with $q<0$, in front of the lens. From the back of the lens, a virtual image appears to exist at some location in front of the lens, but there is not actually an image at that location. However, rays can still be magnified and reproduced to form a real image further along the optical axis, in back of the lens.



For an object in front of the lens, the image falls into the following three ranges:

1) $p>2f$, image real, $M<1$, $2f>q>f$, demagnified, inverted

2) $2f>p>f$, image real, $M>1$, $q>2f$, magnified, inverted

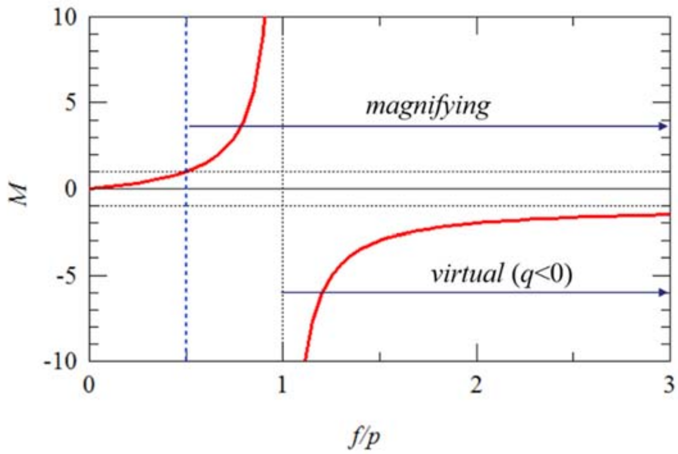
3) $f>p>0$, image virtual, $M<-1$, $q<0$ magnified, not inverted

Lateral magnification (II)

If our object distance p is fixed, and we are changing f (turning the focus knob on the TEM), a useful form is

$$M = \frac{1}{\frac{p}{f} - 1}$$

Here we can see that M is positive (image inverted) when the object is outside of the focal point ($p>f$) and negative (image not inverted) when the object is inside the focal point ($f>p$). Also the image is magnified ($|M|>1$) when $p<2f$.



Angular magnification (II)

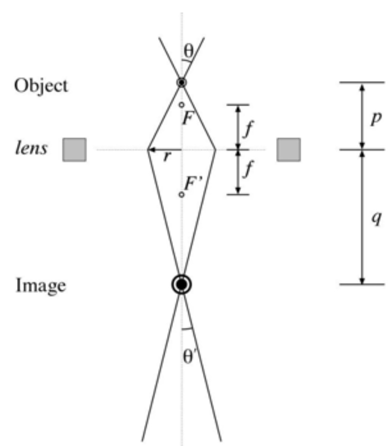
Consider rays diverging from an on-axis point on the object at angle θ from the optic axis towards the lens. The rays cross the lens plane at a radius r, so $\tan\theta = r/p$. These rays must again converge to a point on the image, so on the back side of the lens they make a different angle θ' from the optic axis, so $\tan\theta' = r/q$. The angular magnification is given by

$$M_\theta = \frac{\theta'}{\theta}$$

If we assume small angles

$$M_\theta \approx \frac{\tan\theta'}{\tan\theta} = \frac{p}{q} = \frac{1}{M}$$

So angular magnification is the inverse of lateral magnification.



Source brightness

We often say "brightness" to express something like intensity, and our TEM even has a knob labeled "BRIGHTNESS" that changes the size of the beam spot in the plane of the specimen. But there is a mathematical definition of the brightness of a source of illumination; in this case, electrons. It is a characteristic of the source itself, given by:

$$\text{Brightness} = \frac{\text{Current Density}}{\text{Solid Angle}}, \text{ or } \beta = \frac{j}{\Omega}$$

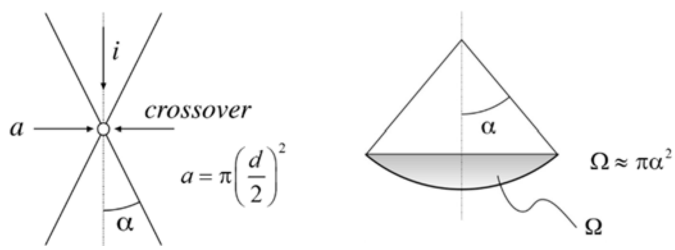
Current density j is electrical current per unit area. Solid angle Ω is the angle by which the beam converges or diverges at a crossover (focus) point. The crossover plane actually contains an image of the original source. Say that image is a roughly uniform, circular disk with area a. If the current in the disk is i, the current density is $j = i/a$. If the beam converges with solid angle Ω, we have

$$\beta = \frac{i}{a \cdot \Omega}$$

where β is the brightness, with units

$$[\beta] = \frac{\text{A}}{\text{cm}^2 \cdot \text{sr}}$$

We usually talk about the diameter (size) d of the probe image, rather than its area a, so for a circular probe $a = \pi \cdot (d/2)^2$. We also usually know the semi-angle of convergence α of the probe, rather than the solid angle. A little trig shows that $\Omega = 2\pi \cdot (1 - \cos\alpha)$, so for small α, $\Omega \approx \pi\alpha^2$.



Conservation of brightness
We can show that β has the same value when measured *at any crossover* along the electron-optical column. If we measure β at the crossover in plane 1, which is just before an aperture and lens combination, we have

$$\beta_1 = \frac{i_1}{a_1 \cdot \Omega_1}$$

The lens focuses the beam to a second crossover in plane 2. The aperture reduces the solid angle on the incident side to some solid angle Ω, thus limiting the current, so the current reaching the crossover at point 2, below the lens, is

$$i_2 = i_1 \frac{\Omega}{\Omega_1}$$

The area of the crossover spot varies as its diameter squared, so

$$a_2 = M^2 a_1$$

But the solid angle varies as the angular magnification squared, so

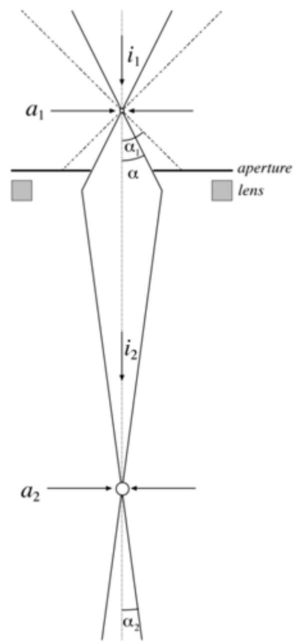
$$\Omega_2 = \frac{\Omega}{M^2}$$

Now let’s find the brightness at point 2:

$$\beta_2 = \frac{i_2}{a_2 \cdot \Omega_2} = \frac{\left(i_1 \dfrac{\Omega}{\Omega_1}\right)}{\left(M^2 a_1\right) \cdot \left(\dfrac{\Omega}{M^2}\right)} = \frac{i_1}{a_1 \cdot \Omega_1} = \beta_1$$

We see the the brightness is the same for the two focused probes. It is a characteristic of the source, regardless of how we form its image.

1:



Electron sources
There are two main types of electron sources: 1) thermionic and 2) field-emission. Thermionic sources emit electrons by virtue of high temperature (T). Field-emission sources emit electrons by virtue of high electric field (E).

The current from a thermionic source is given by:

$$j = AT^2 \exp\left(-\frac{\Phi_w}{kT}\right)$$

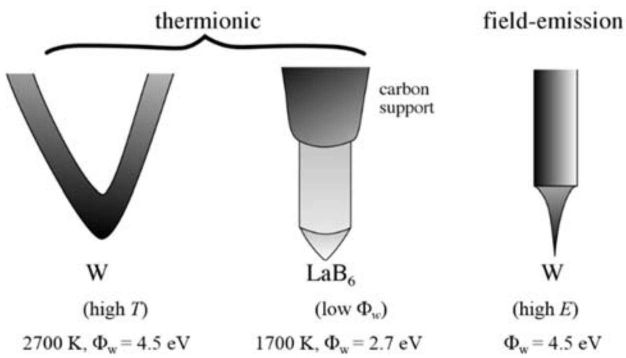
Here A is called the Richardson’s constant for the material and Φ_w is the work function: the minimum energy to transfer an electron from equilibrium in the material into the vacuum:

The current from a field-emission source results from quantum-mechanical tunneling of electrons through the potential barrier presented by the work function:

$$j = \frac{AE^2}{\Phi_w} \exp\left(-\frac{B\Phi_w^{1.5}}{|\mathrm{E}|}\right)$$

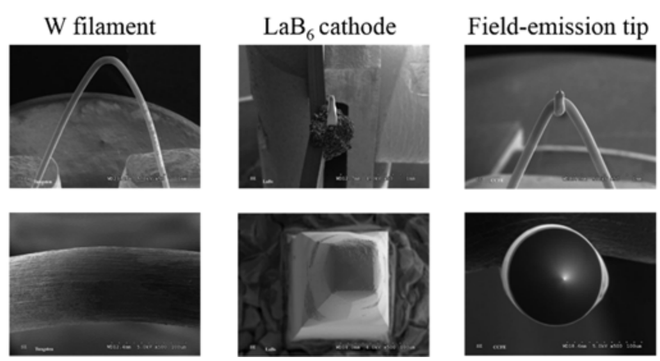
There are only a few examples of each of these that are used in almost every TEM and SEM. Thermionic sources are almost always bent wires of tungsten (W) or LaB_6 crystals mounted to a wire of refractory metal (maybe W) with a carbon support. In the case of W, we correctly call this a “filament”, because a current is passed through it to heat it resistively, and part of this current is emitted thermionically. Strictly speaking, it is not correct to call a LaB_6 source a filament, because it is heated indirectly by thermal conduction through the carbon support. Instead, we call the LaB_6 source a “cathode”. (But we my call the wire that heats it a filament.)

A field-emission source does not have to be hot at all, but it does have to be very sharp for a high electric field to be established around it. Because the tip is so small, it is practically a point source of electrons, so the emission has very high coherence, which has advantages for some techniques.



We often call any of these sources, along with their housing, the “gun”.

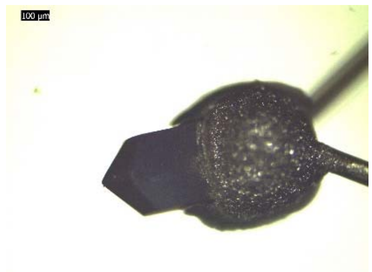
Electron sources
The geometry of these different types of sources are shown below in SEM images (courtesy M. Grimson).



Brightness of electron sources
The approximate brightnesses of these sources are listed below. W filaments are very inexpensive ($50), but they have a short life, maybe a few hundred hours. They are good for low-mag work where uniform illumination is needed. LaB_6 cathodes are quite expensive, usually around $1K. They have long life if used carefully, and are best for medium to high magnification TEM work. Field emission sources are the most expensive, maybe $3K. They are the best for very high-mag work, especially using STEM.

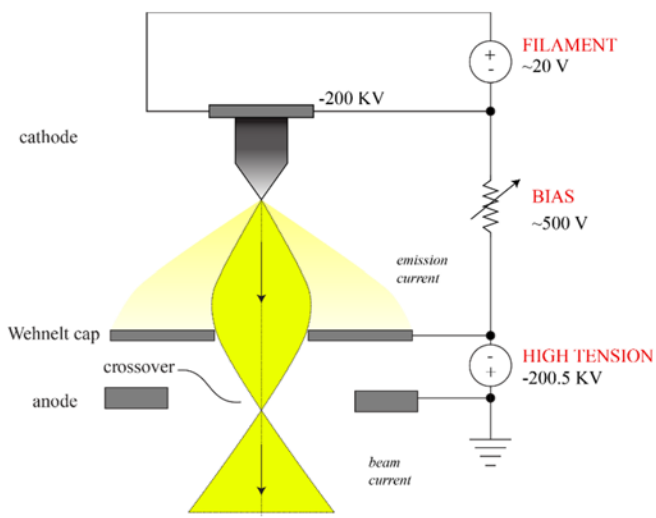
Source Type	Brightness $[A/(cm^2 \cdot sr)]$
W	10^4
LaB_6	10^5
Field Emission	10^7

Degradation of the source can usually be observed in an optical microscope. An optical image of a LaB_6 crystal after 720 h of use is shown below.



Triode gun

A thermionic electron source is usually in a triode configuration, meaning that it is part of a three-terminal electrical circuit. The gun is connected to the high-voltage cathode (negative) terminal with respect to the anode (ground), which accelerates the beam electrons into the microscope column. But a third connection is made to an electron valve, called the Wehnelt cap. This is a metal shield around the gun with a small aperture in the center that sits very close to the cathode tip, between the cathode and anode. The voltage on the Wehnelt, called the "bias", along with its distance from the tip, allow additional control of the beam current, as well as focusing to form a demagnified image of the tip. The cap is a type of electrostatic lens, which creates a potential barrier in the range of a few hundred volts that pushes electrons towards the optical axis.



The actual current that heats the filament is driven by a fairly small (tens of V) DC voltage riding on top of the accelerating voltage. The bias voltage is usually created by a variable resistor between the cathode and Wehnelt. But, some current is collected by the Wehnelt, which causes the bias voltage to vary with the emission current. As the emission current increases, the bias increases, damping out further increases in emission current. This is called a self-biasing configuration. It helps to protect the filament and stabilize the beam current.