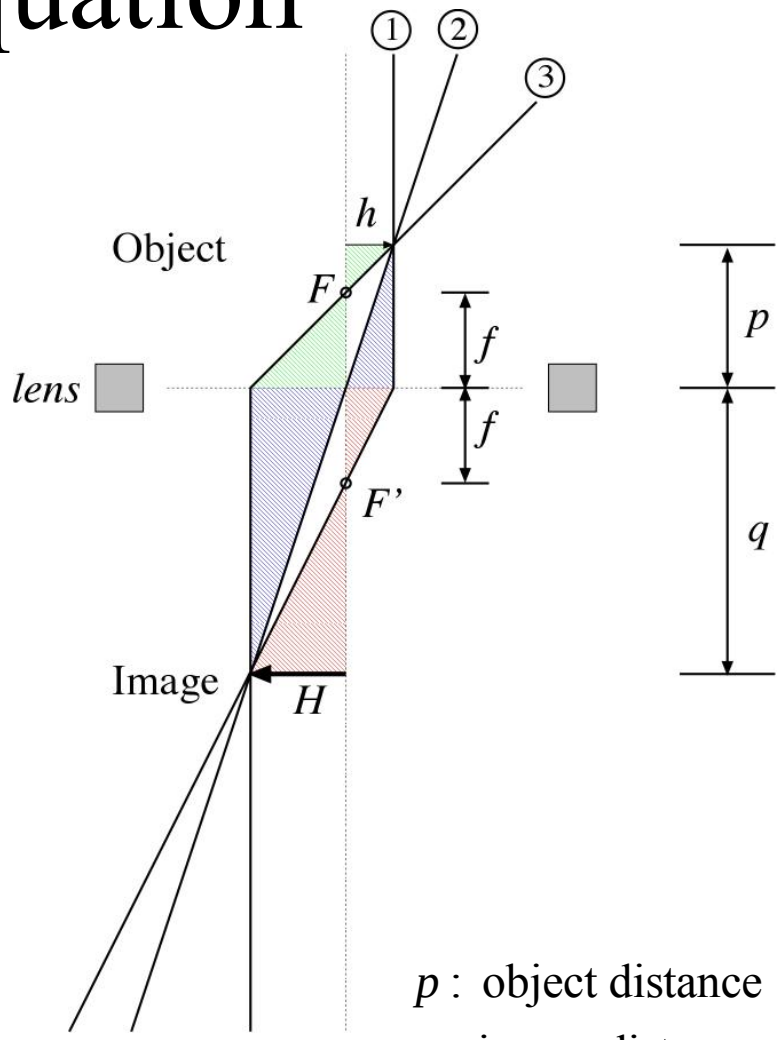
Lens equation

Equate Angles of Similar Triangles:

$$\frac{h}{f} = \frac{H}{q-f} \qquad \frac{h}{p} = \frac{H}{q} \qquad \frac{h}{p-f} = \frac{H}{f}$$

1 2 3

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$



Relates Object & Image Positions to Focal Length.

p : object distance

q : image distance

f : focal length

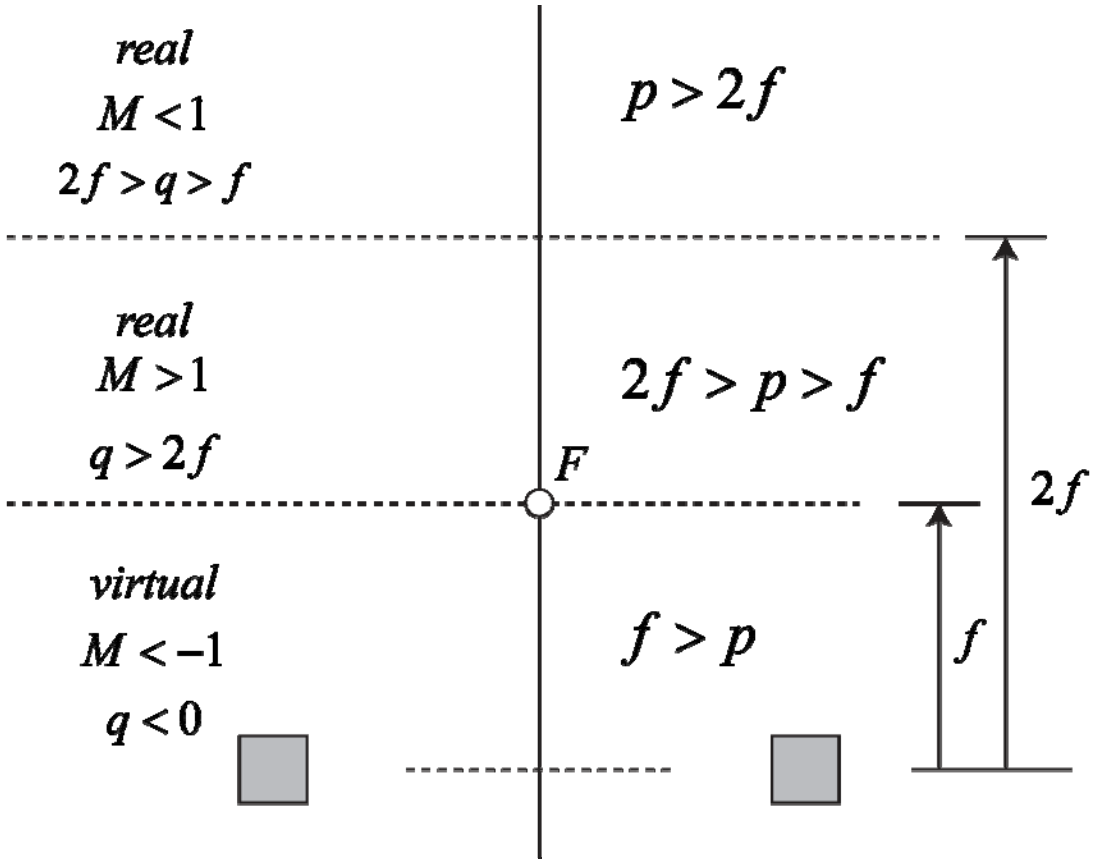
Lateral Magnification (I)

$$M = \frac{H}{h} = \frac{q}{p}$$

$$q = \frac{1}{\frac{1}{f} - \frac{1}{p}}$$

$q < 0$: virtual

$q > 0$: real



$|M| > 1$ magnified

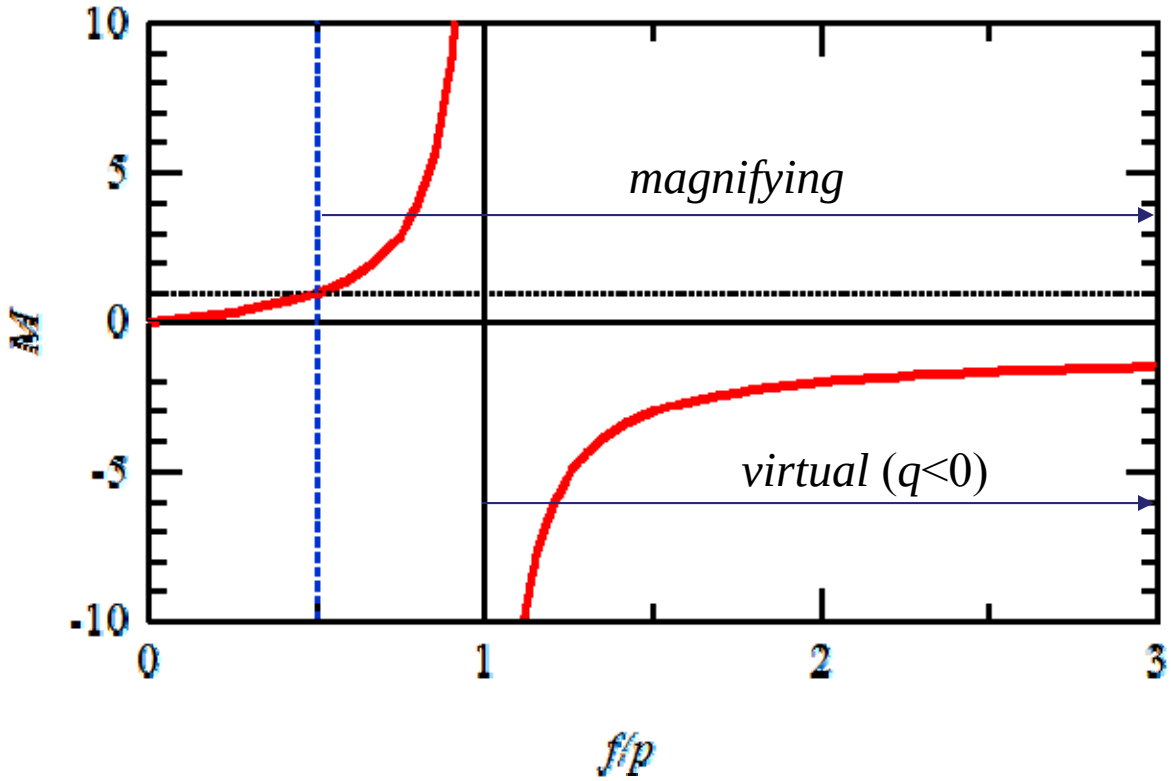
$0 < |M| < 1$ demagnified

Lateral Magnification (II)

$$M = \frac{H}{h} = \frac{q}{p} = \frac{1}{\frac{p}{f} - 1}$$

$M < 0$: not inverted

$M > 0$: inverted



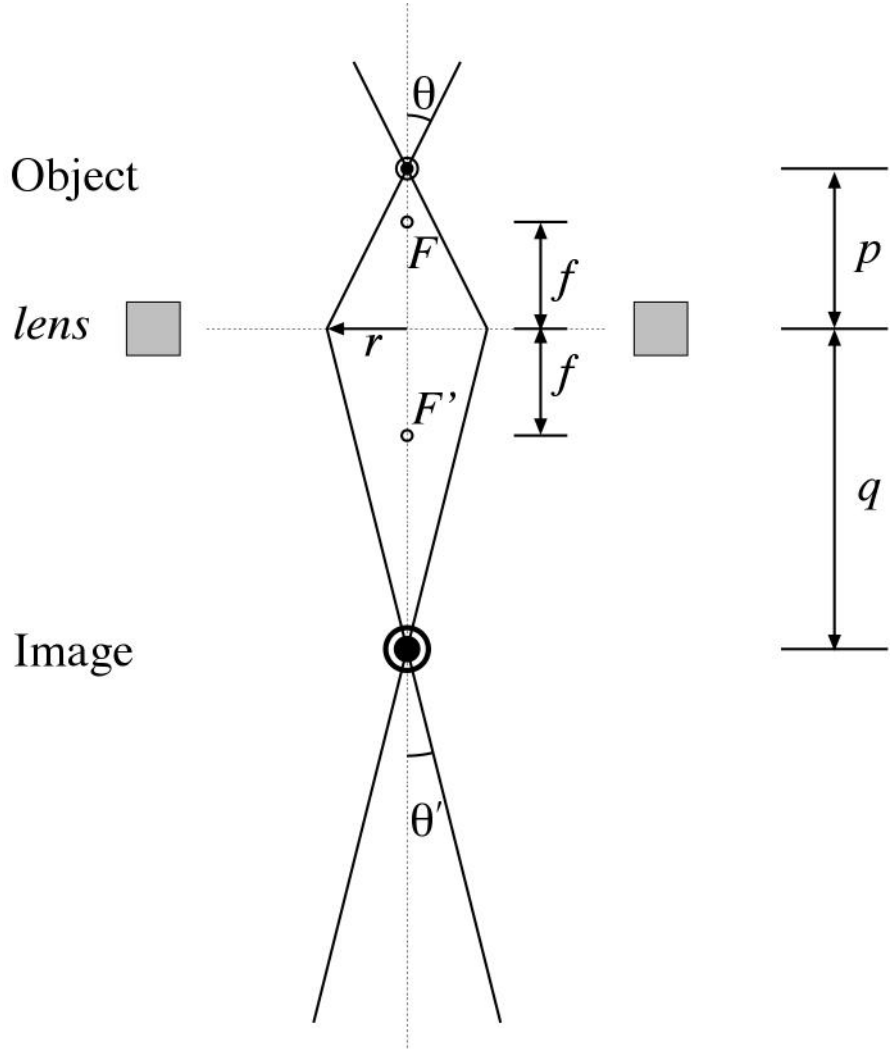
Angular Magnification

Consider small object, on axis

$$\tan\theta = \frac{r}{p} \qquad \tan\theta' = \frac{r}{q}$$

$$M_\theta = \frac{\theta'}{\theta} \approx \frac{\tan\theta'}{\tan\theta} = \frac{p}{q} = \frac{1}{M}$$

inverse of lateral mag.



Source Brightness

Characteristic of source

$$\text{Brightness} = \frac{\text{Current Density}}{\text{Solid Angle}}$$

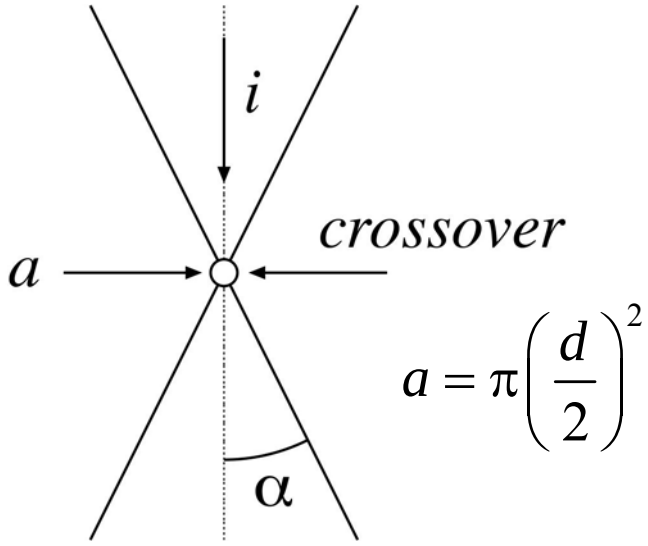
i: current

a: probe area

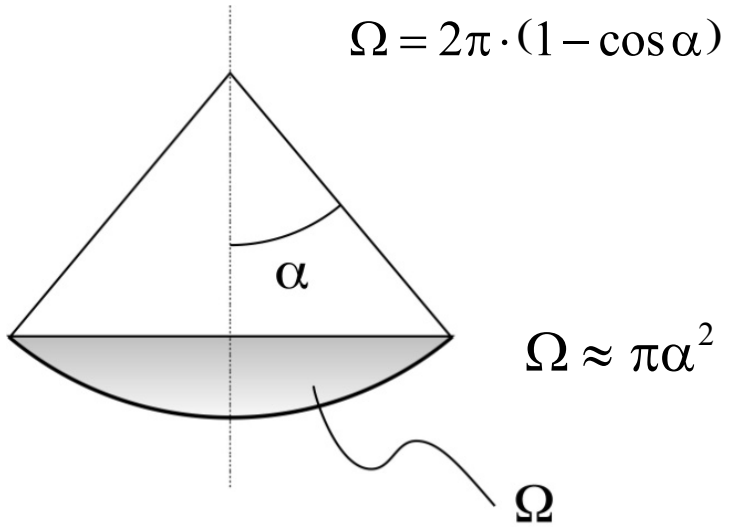
Ω: solid angle

$$[\beta] = \frac{\text{A}}{\text{cm}^2 \cdot \text{sr}}$$

$$\beta = \frac{i}{a \cdot \Omega} \qquad \text{//brightness}$$



$$a = \pi\left(\frac{d}{2}\right)^2$$



$$\Omega = 2\pi \cdot (1 - \cos\alpha)$$

$$\Omega \approx \pi\alpha^2$$

Conservation of Brightness

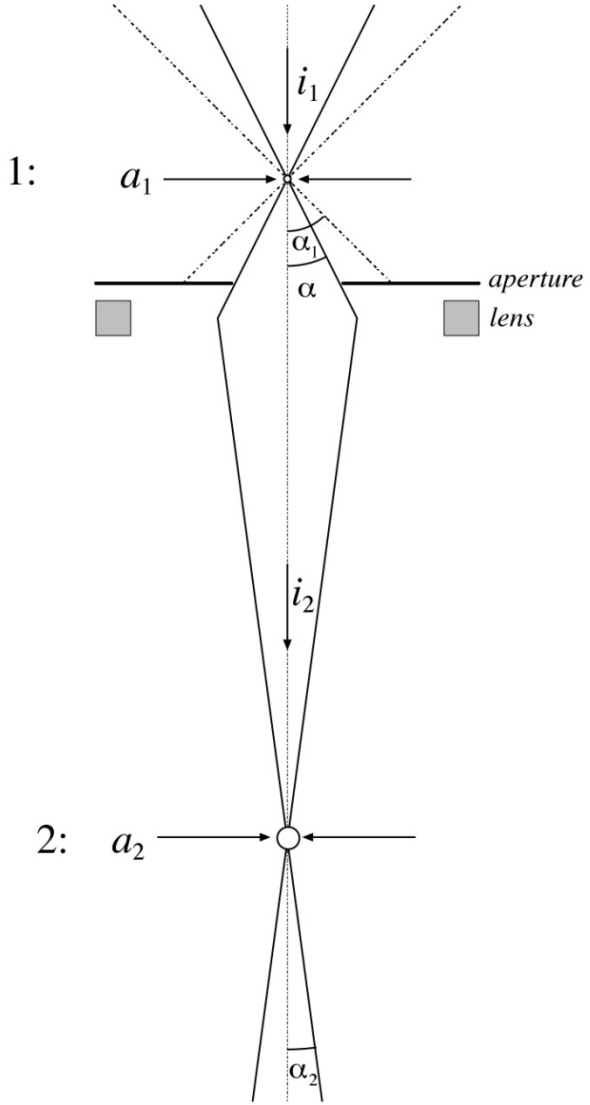
Same β in every plane along path

At 1: $$\beta_1 = \frac{i_1}{a_1 \cdot \Omega_1}$$

At 2: $$i_2 = i_1 \frac{\Omega}{\Omega_1}$$

$$a_2 = M^2 a_1 \qquad \Omega_2 = \frac{\Omega}{M^2}$$

$$\Rightarrow \quad \beta_2 = \frac{i_2}{a_2 \cdot \Omega_2} = \frac{\left(i_1 \frac{\Omega}{\Omega_1} \right)}{\left(M^2 a_1 \right) \cdot \left(\frac{\Omega}{M^2} \right)} = \frac{i_1}{a_1 \cdot \Omega_1} = \beta_1$$



Electron Sources

Two main types:

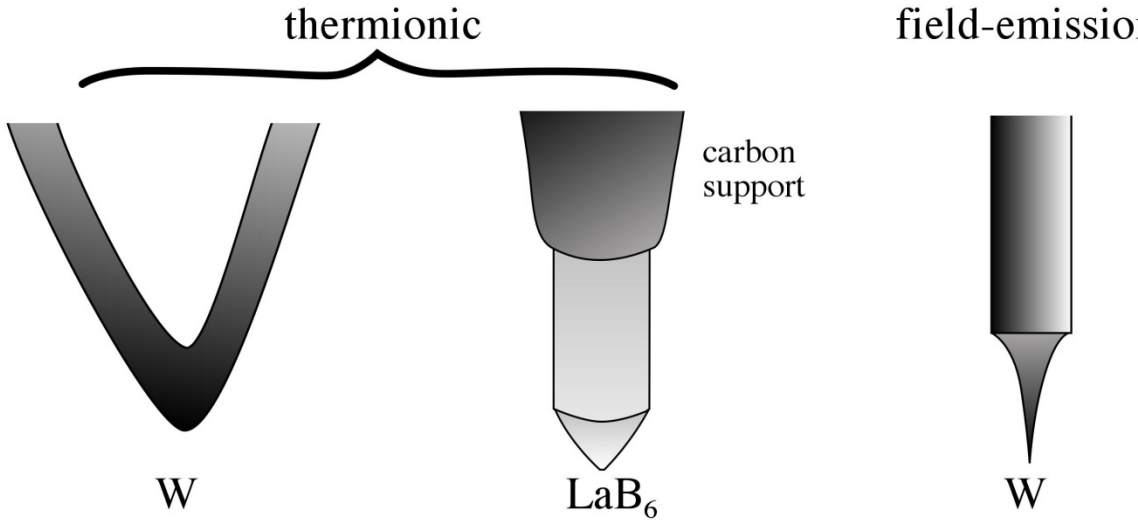
1) Thermionic
   High temperature (T)

2) Field-Emission
   High electric field (E)

$$j = AT^2 \exp\left(-\frac{\Phi_w}{kT}\right)$$

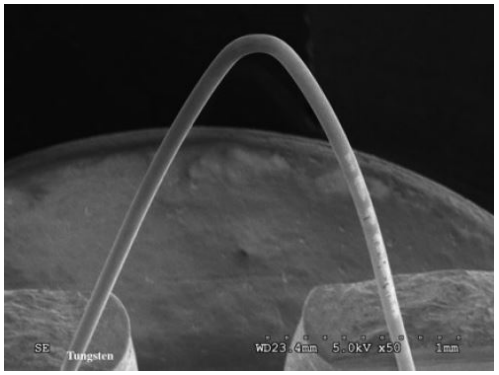
$$j = \frac{AE^2}{\Phi_w} \exp\left(-\frac{B\Phi_w^{1.5}}{|E|}\right)$$

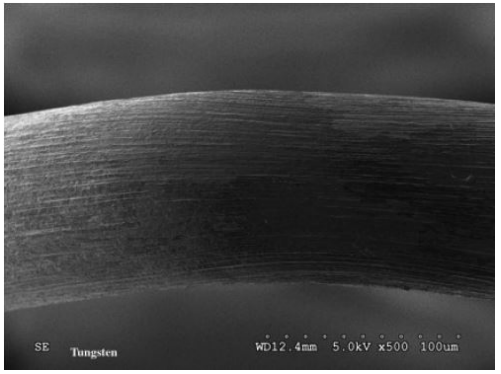
Φ_w : work function



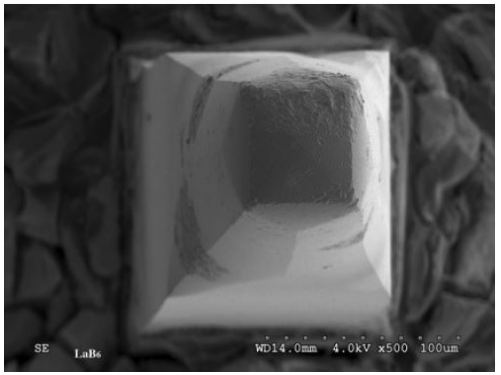
Electron Sources

W filament

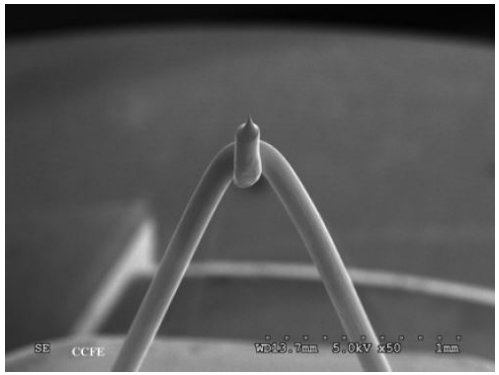


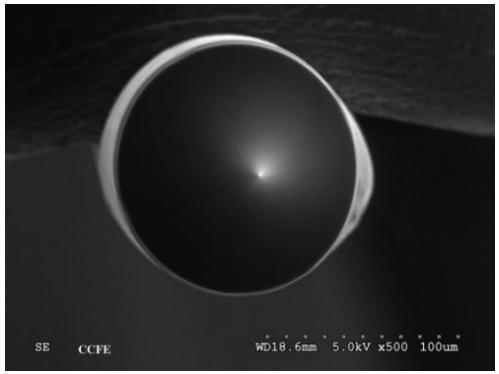


LaB_6 cathode



Field-emission tip

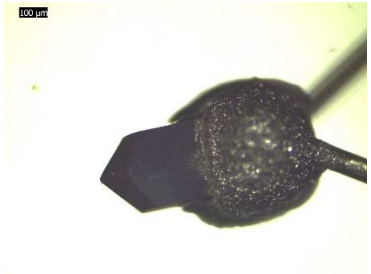




M. Grimson, online.

Brightness of electron sources

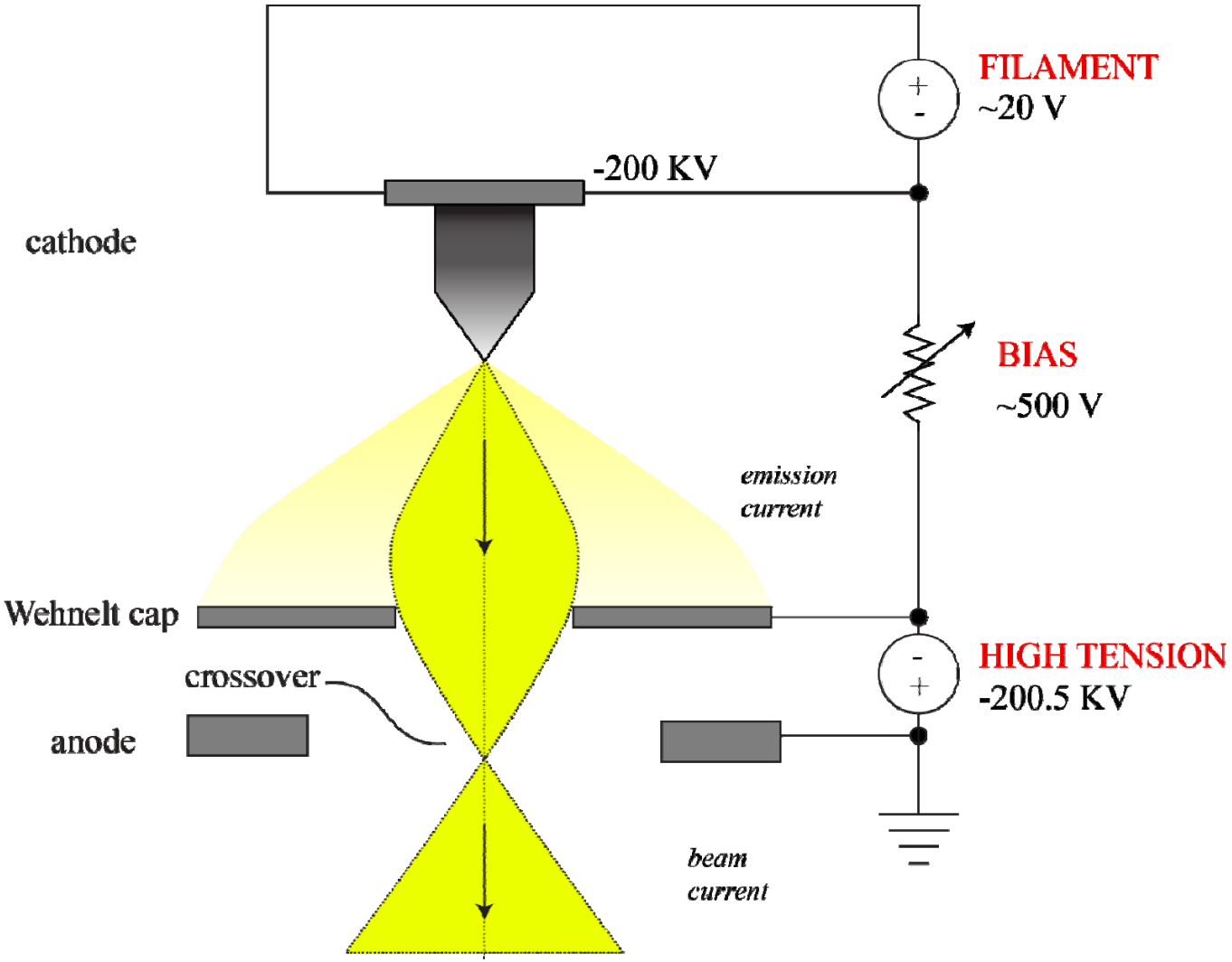
Our LaB_6 after 720 h of use:



Source Type	Brightness [$A/(cm^2 \cdot sr)$]
W	10^4
LaB_6	10^5
Field Emission	10^7

W: inexpensive ($20), short-life, good for low-mag work
LaB_6: expensive ($1K), long life, good for medium-mag work
FE: very expensive ($3K), good for high-mag work

Triode gun



Wehnelt cap forms a demagnified image of the source