NANO 703-Notes

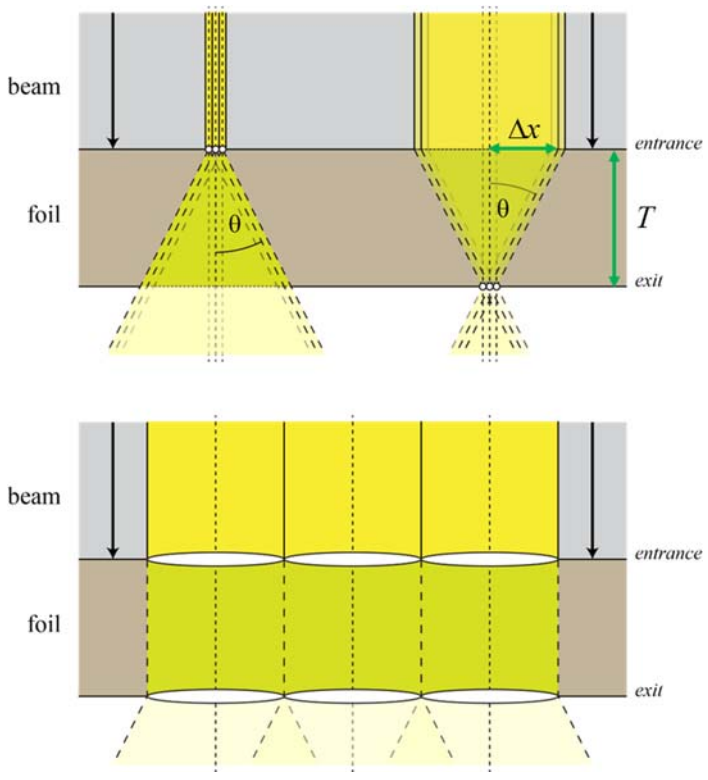
Chapter 24: Thickness and Bending

Column approximation

When describing the appearances of TEM images, we usually start by assuming the incident is a perfect plane wave moving directly downward along z. Let's also say our foil has uniform thickness T. After an electron enters the sample at some point on its top (entrance) surface, it begins to scatter out to different angles. Maybe significant scattering only contributes out to some angle θ. Then the electron while interact with a volume of material contained in a cone of semi-angle θ. After passing throught he exit surface of the foil, the electron wave function remains distributed over this cone of scattering directions.

A point on the bottom (exit) surface is exposed to scattering from any of the points on the top surface contained within the circle inscribed by an inverted cone of the same semi-angle θ. So if we want to precisely calculate the exit wave function at this point, we need to combine contributions from this entire cone. If the sample is perfectly uniform, this is easy, because scattering at any point is the same as that any other point. But if we are trying to predict how some small, localized feature will look, the calculation will be more complicated.

How big is the region on the entrance surface that contributes to the wave function at some point on the exit surface? For small angles, its radius is $\Delta x = T \cdot \theta$. Say the thickness is $T = 100$ nm and $\theta = 10$ mrad (a typical small angle). Then points within a radius $\Delta x = 1$ nm on the entrance surface influence a point on the exit surface.



The column approximation assumes that the cone is actually a narrow cylinder. As long as the feature we are analyzing varies on a length scale larger than $2\Delta x$, we can just compute the exit wave function point-by-point along the sample. Two adjacent cylinders separated by more than $2\Delta x$ do not influence one another.

Example: Two-beam, dark-field image contrast

If we are just trying to explain image features on a fairly coarse length scale (larger than Δx), the column approximation works just fine. For example, if we have a thin foil, and we want to calculate the image contrast near a two-beam condition, we can start with the the diffracted intensity we already know:

$$I_{\mathbf{g}} = \left(\frac{\pi T}{\xi}\right)^2 \cdot \mathrm{sinc}^2\left(\pi s_{eff} T\right)$$

where

$$s_{eff} \doteq \sqrt{s^2 + \left(\frac{1}{\xi^2}\right)}$$

It is often that case that either s, ξ, or T are slowly varying across the field of view. This is a pretty good time to use the column approximation:

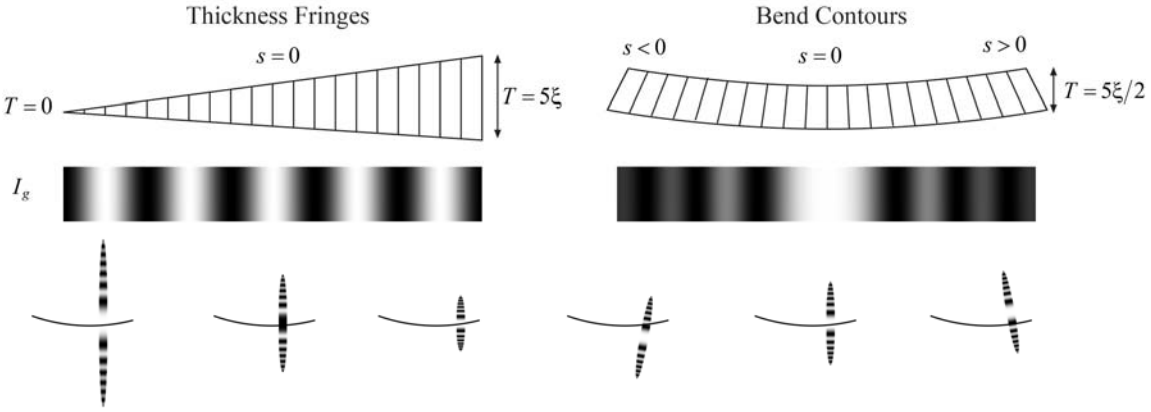
$$I_{\mathbf{g}}(x) \approx \left[\frac{\pi T(x)}{\xi(x)}\right]^2 \cdot \mathrm{sinc}^2\left[\pi s_{eff}(x) \cdot T(x)\right]$$

where $s(x)$, $\xi(x)$, and $T(x)$ are evaluated locally at x, which is a lateral coordinate measured parallel to the image plane.

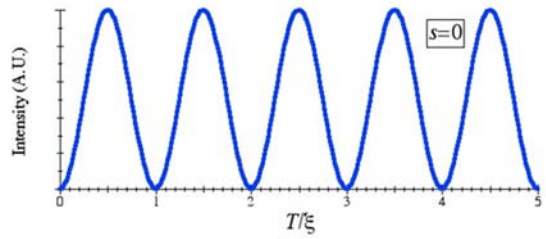
Now, if there are variations along the beam direction, we will need to analyze those using from scratch, using theoretical methods, such as Bloch waves. But if the variations of interest are all lateral, the column approximations will suffice.

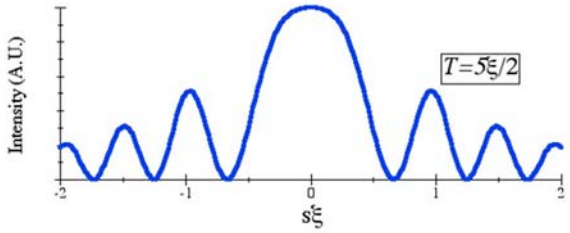
Thickness fringes & bend contours

Two common situations where the column and two-beam approximations combined give us a fairly complete description of the resulting image intensity are thickness fringes and bend contours. Say the extinction distance ξ is constant. Thickenss fringes arise when we are at or near a two-beam condition and the thickness is varying. At precisely $s = 0$, we expect the intensity to go through one full intensity oscillation for every change by ξ in thickness. In the kinematical picture (which is always a bit shaky near the Bragg condition) the center of the relrod, where it is intersected by the Ewald sphere, may be either an intensity minimum or maximum, depending on thickness.



If the foil is bent, it is the excitation error that varies from point to point. Now the whole relrod is higher or lower, and tilted one way or the other, depending on the local crystal orientation. So the point where the relrod interesects the Ewald sphere oscillates between light and dark across the field of view.





Example: Si

An example of thickness fringes and bend contours in a 220 DF image of a silicon plan-view specimen are shown below. Some additional sharp features appear to arise from HOLZ diffraction.