Scattering/Wave Terminology

elastic/inelastic (particle properties, collisions):
elastic - kinetic energy conserved
inelastic - some change (loss) of kinetic energy

coherent/incoherent (wave properties):
coherent - constant phase relationship
incoherent - no distinct phase relationship

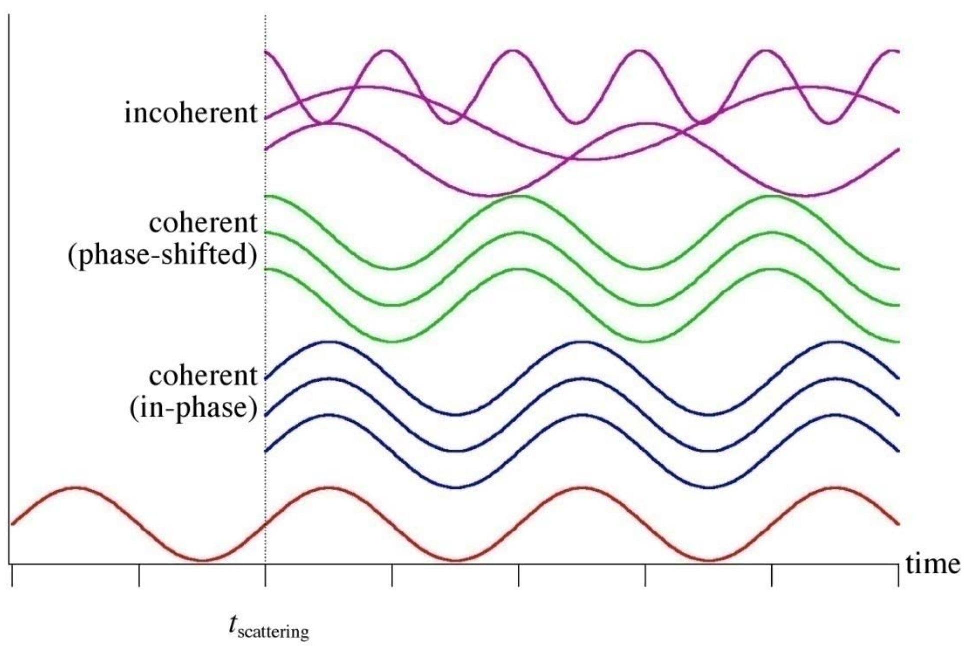
scattering angle (change of direction):
forward - less than 90°
back - greater than 90°

Coherent Scattering

-scattering changes electron phase by a constant amount (possibly zero)

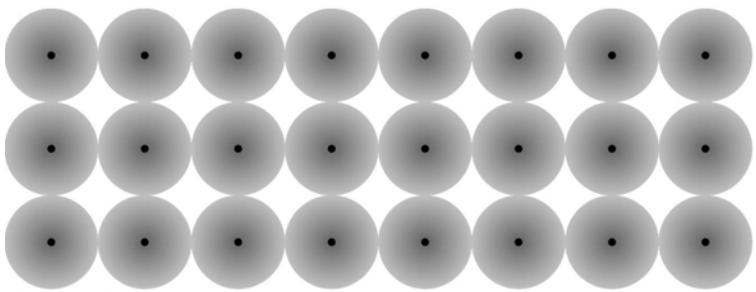
-scattering is elastic: no change in electron wavelength

$$E \rightarrow p \rightarrow \lambda$$

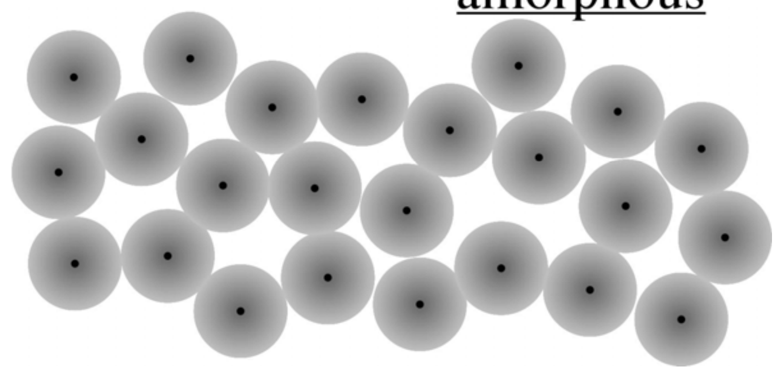


Coherent vs. Incoherent Solids

crystalline



amorphous

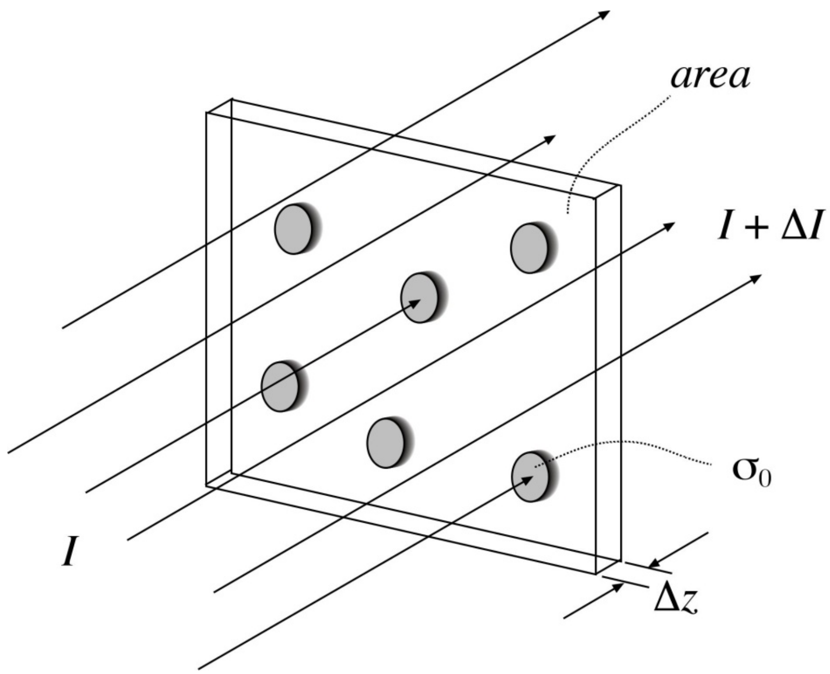


Long-range order

No long-range order

May have short-range order

Scattering Cross-Section



n: # of scatterers/volume

Thin Slab:

N: # of scatterers

$$N = n \cdot area \cdot dz$$

σ_0: cross-section (area) of one scatterer

$$[\sigma_0] = \text{barns; } 1 \text{ b} = 10^{-24} \text{ cm}^2$$

$\frac{N \cdot \sigma_0}{area}$: fraction of incident beam scattered

$\Rightarrow$ Intensity change through a ***very thin*** slice

$$dI = -n \cdot \sigma_0 \cdot dz \cdot I$$

Mean Free Path

$$\frac{dI}{I} = -\mu \cdot dz$$

//Fractional intensity change through thin slice

$$n = \frac{N_A \cdot \rho}{A}$$

//Assume a single element

$$\mu = n \cdot \sigma_0 = \frac{N_A \cdot \rho \cdot \sigma_0}{A}$$

//Attenuation coefficient (length^{-1})

$N_A = 6.022 \times 10^{23} / \text{mole}$: Avogadro's #

A: molar atomic mass (g/mole)

ρ: mass density (g/volume)

$\rho \cdot T$: mass-thickness

$$\int_{I'=I_0}^{I} \frac{dI'}{I'} = -\mu \cdot \int_{z=0}^{T} dz$$

//Integrate over thickness

$$\boxed{I(T) = I_0 \cdot e^{-\mu \cdot T}}$$

//Correct expression for intensity

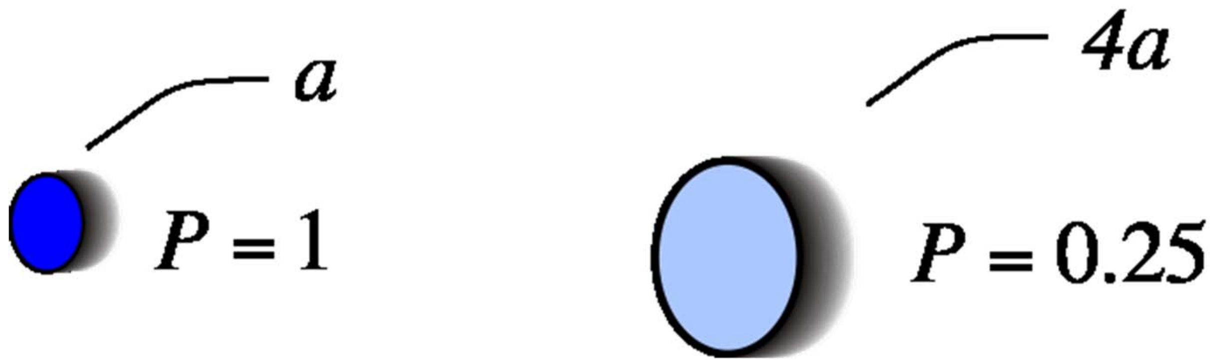
$$\Lambda = \frac{1}{\mu}$$

//Mean free path (length)

Interpreting Cross-Section

Area

Scattering
Probability

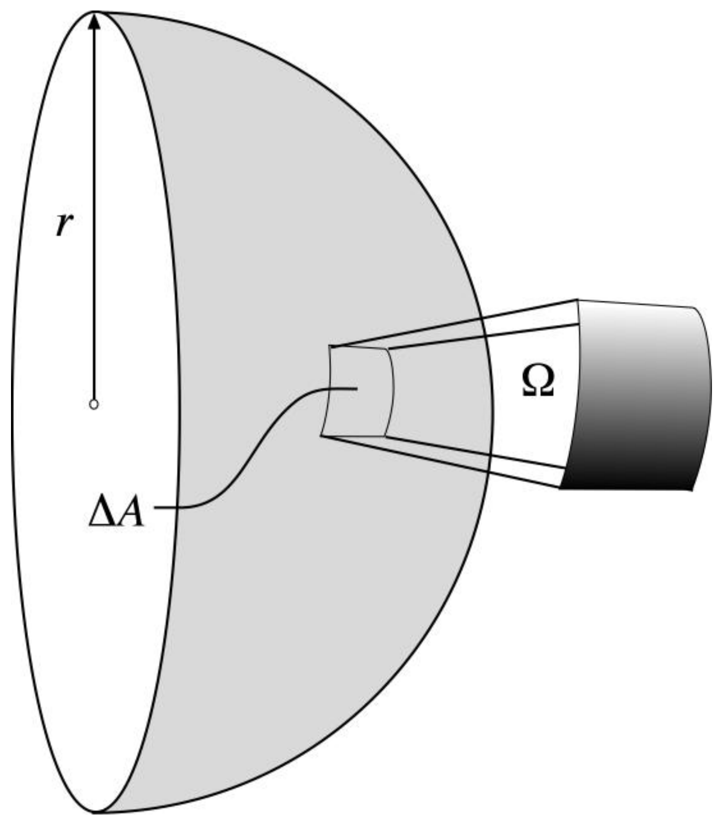


$$\sigma = 1 \cdot a = a \qquad\qquad \sigma = 0.25 \cdot (4a) = a$$

Same total cross-section

Units: *Solid Angle*

surface area of sphere $= 4\pi r^2$



Ω: solid angle

$$\Omega = \frac{\Delta A}{r^2}$$

in steradians: sr

$$0 \leq \Omega \leq 4\pi \text{ sr}$$

Also, $1 \text{ sq. deg.} = \left(\frac{\pi}{180}\right)^2 \text{ sr}$ *sr preferred!*

Scattering from Single Slit (Narrow)

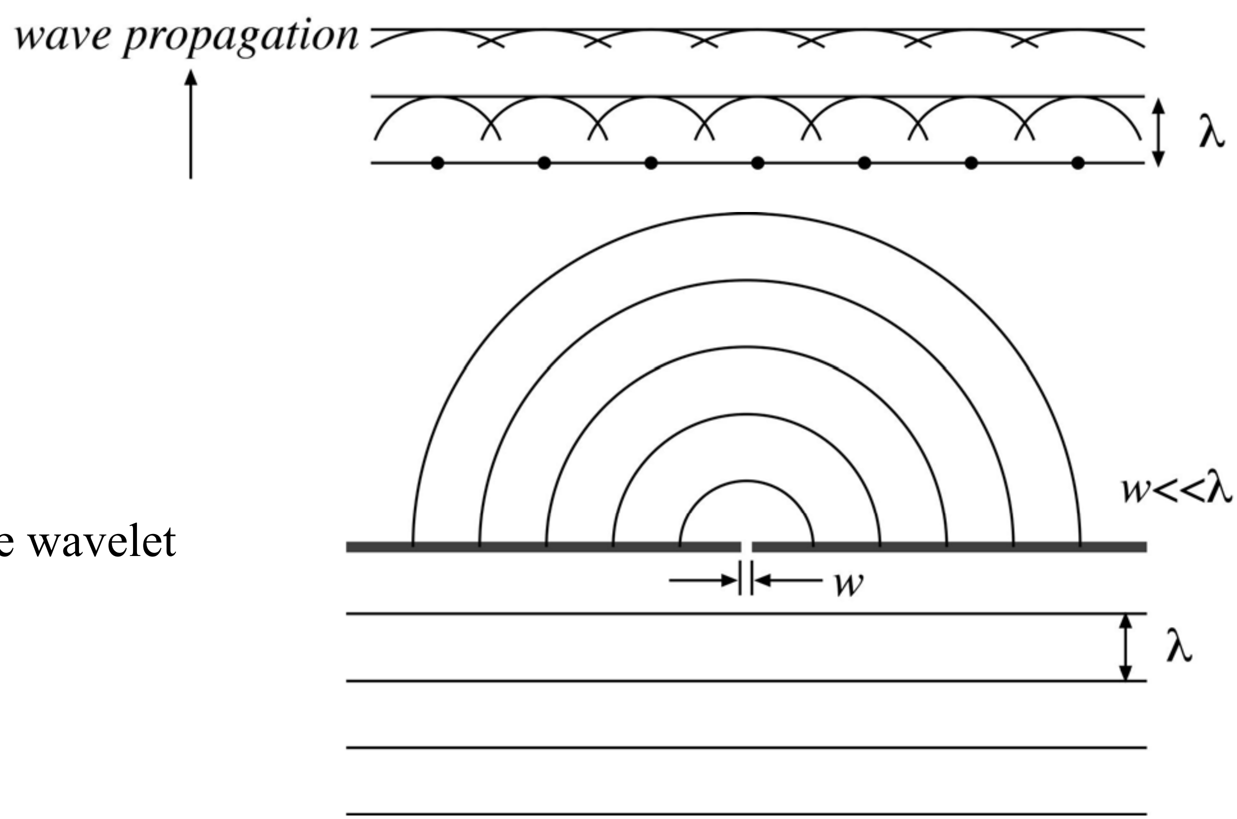
Huygens' Principle: *Every point on a wave front acts as a source of secondary, spherical "wavelets"*

Plane Wave

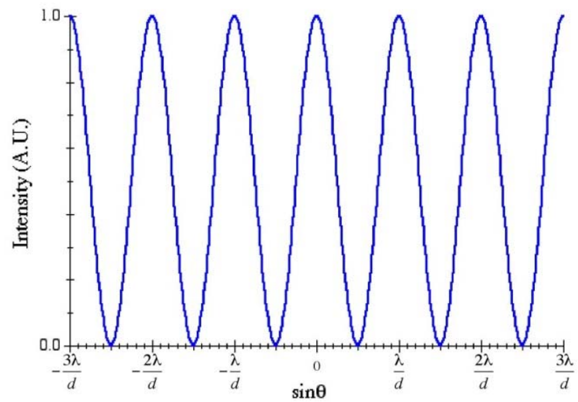
Spherical wavefronts

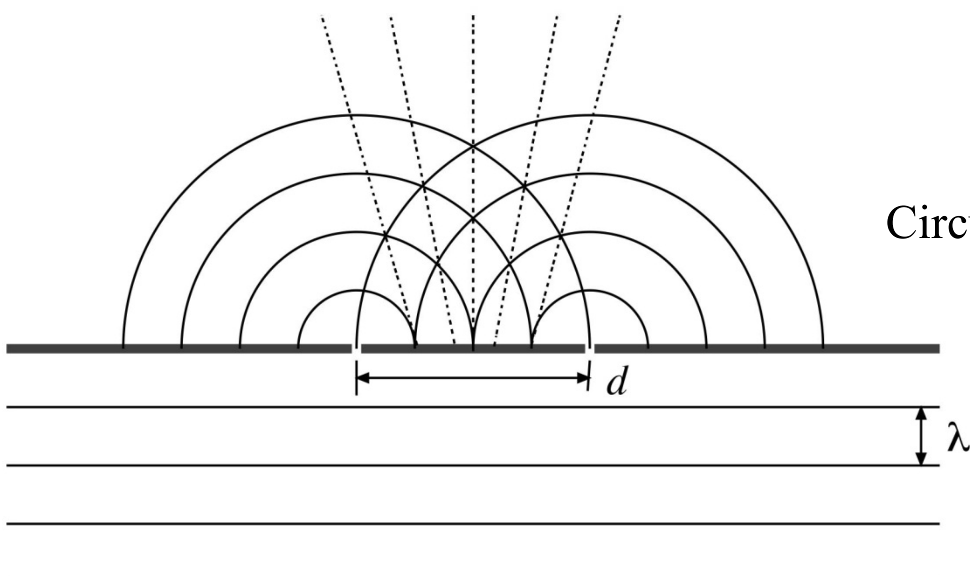
Narrow slit selects one wavelet

→spherical wave



Two-Slit Interference Pattern (Narrow Slits)





Path-length difference gives interference

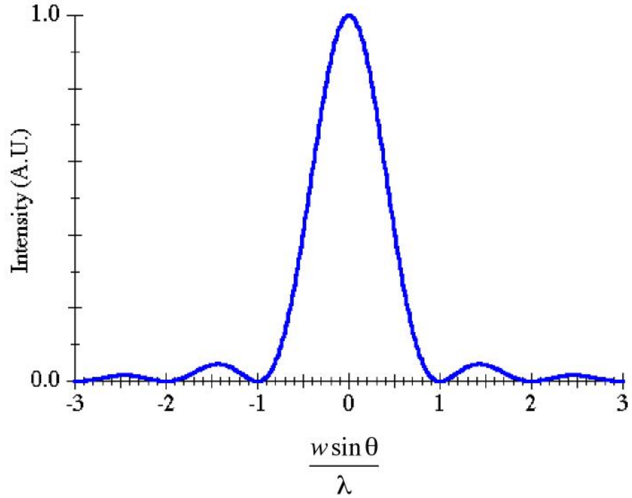
Far-field (Fraunhofer) pattern

$$I(\theta) = I_0 \cdot \cos^2\left(\frac{\pi \cdot d \cdot \sin\theta}{\lambda}\right)$$

Circular wavelets from narrow slits (small *w*)

http://www.phy.ntnu.edu.tw/ntnujava/index.php?topic=15

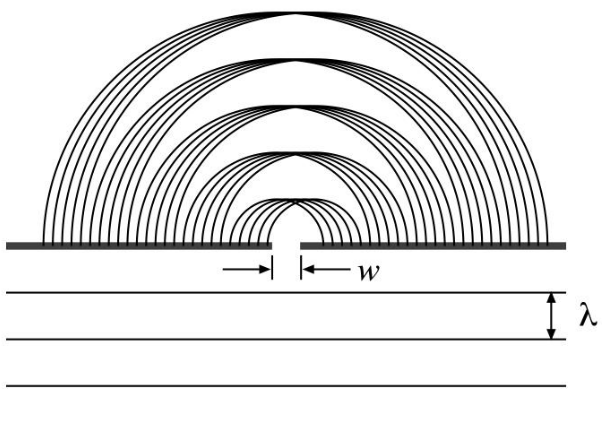
Single-Slit Diffraction



Bright, central maximum

$$I(\theta) = I_0 \cdot \mathrm{sinc}^2\left(\frac{\pi \cdot w \cdot \sin\theta}{\lambda}\right)$$

Fringes



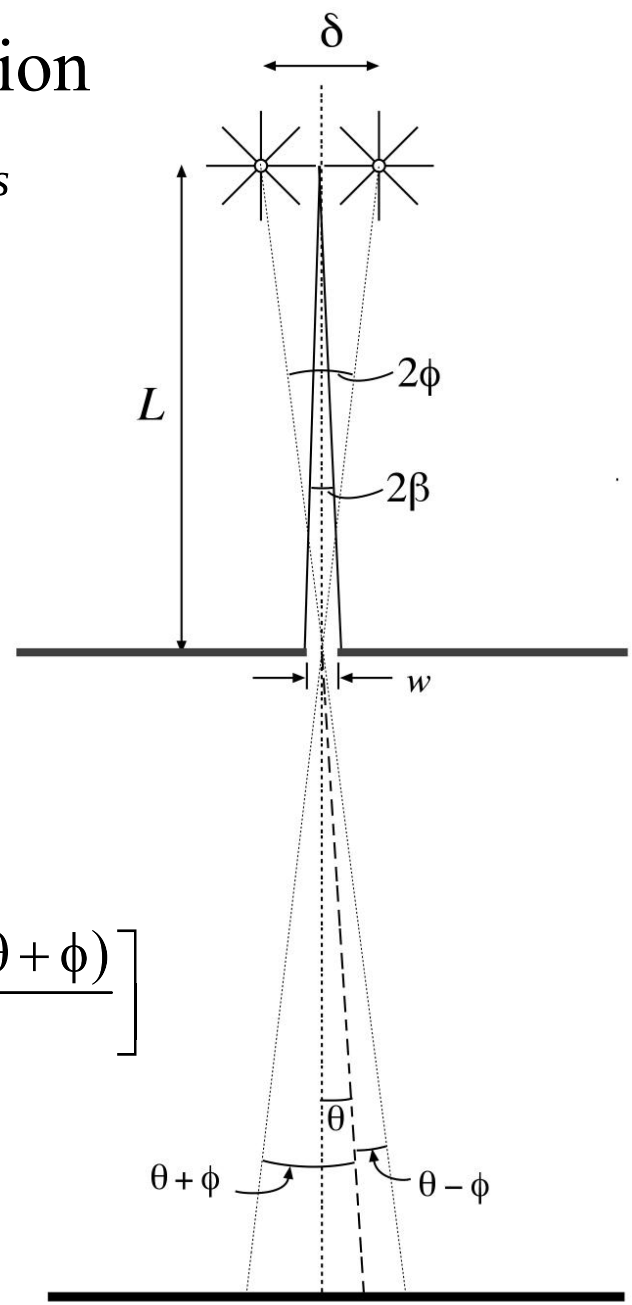
$$\mathrm{sinc}(x) \equiv \frac{\sin x}{x}$$

Small angles:

$$\lambda \ll w \Rightarrow \sin\theta \approx \theta$$

Two-Slit Superposition

Assume incoherent sources



β: semi-angle of collection

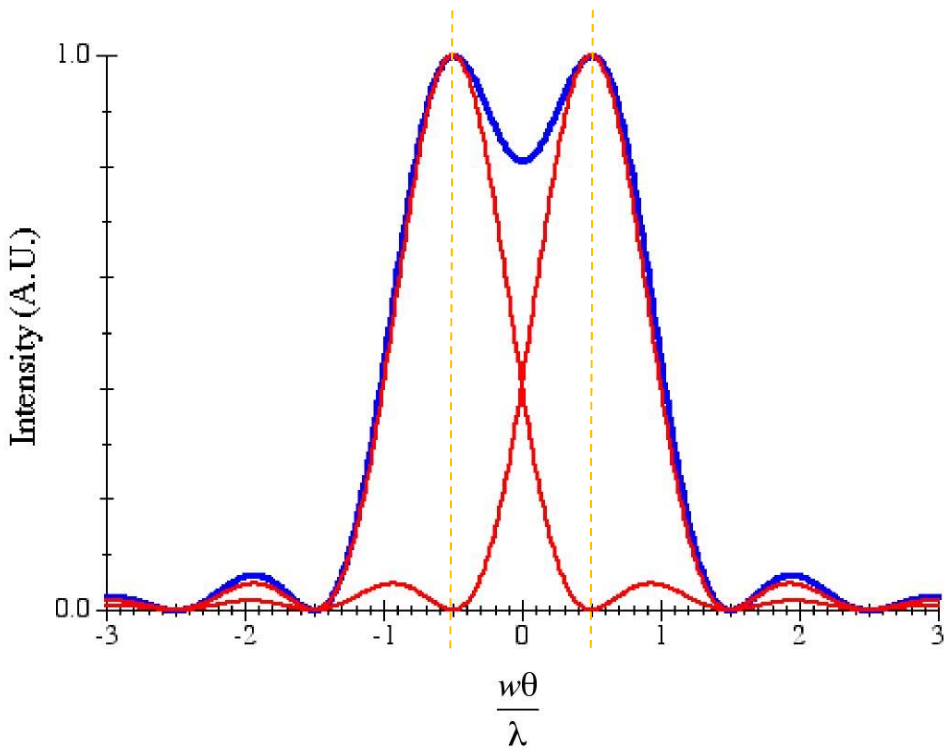
ϕ: semi-angle between sources

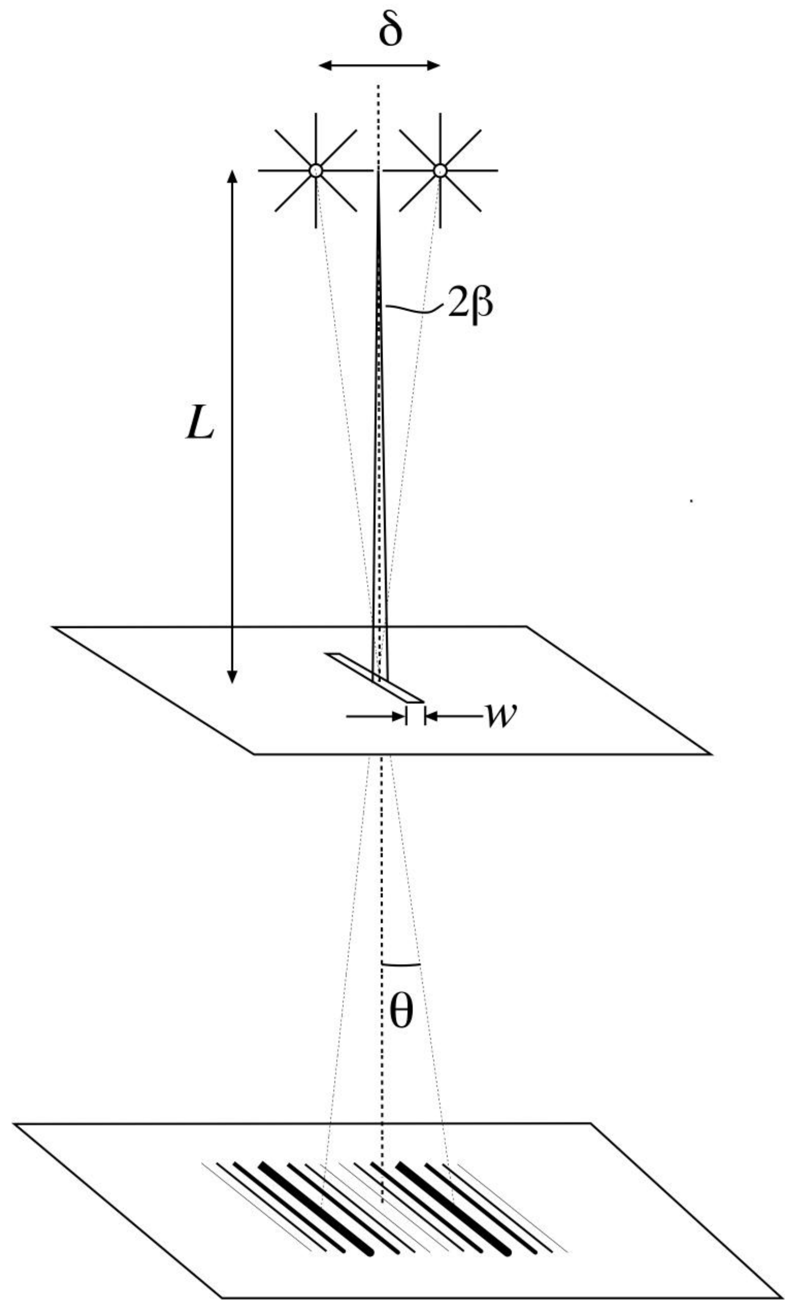
Small angles: $\beta \approx \dfrac{w}{2L}$ $\phi \approx \dfrac{\delta}{2L}$

$$I(\theta) = I_0 \cdot \mathrm{sinc}^2\left[\frac{\pi \cdot w \cdot (\theta - \phi)}{\lambda}\right] + I_0 \cdot \mathrm{sinc}^2\left[\frac{\pi \cdot w \cdot (\theta + \phi)}{\lambda}\right]$$

Rayleigh Criterion

Rayleigh criterion:
Just resolvable if central maximum from slit 1 falls outside first minimum from slit 2





Resolution: Rayleigh Criterion for Slits

$$\mathrm{sinc}^2\left[\frac{\pi w(\theta_0 - \phi_{\min})}{\lambda}\right] = 1 \qquad \text{(source 1 max)}$$

&

$$\mathrm{sinc}^2\left[\frac{\pi w(\theta_0 + \phi_{\min})}{\lambda}\right] = 0 \qquad \text{(source 2 min)}$$

$$\mathrm{sinc}(0) = 1$$

&

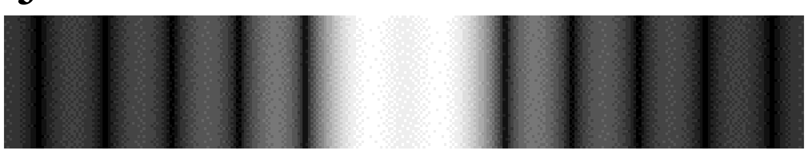
$$\mathrm{sinc}(\pi) = 0$$

$$\frac{w(\theta_0 - \phi_{\min})}{\lambda} = 0 \;\; \& \;\; \frac{w(\theta_0 + \phi_{\min})}{\lambda} = 1$$

$$\frac{2w\phi_{\min}}{\lambda} = 1 \Rightarrow \phi_{\min} = \frac{\lambda}{2w}$$

$$\phi_{\min} = \frac{\delta_{\min}}{2L} \Rightarrow \delta_{\min} = \frac{L\lambda}{w}$$

just resolvable

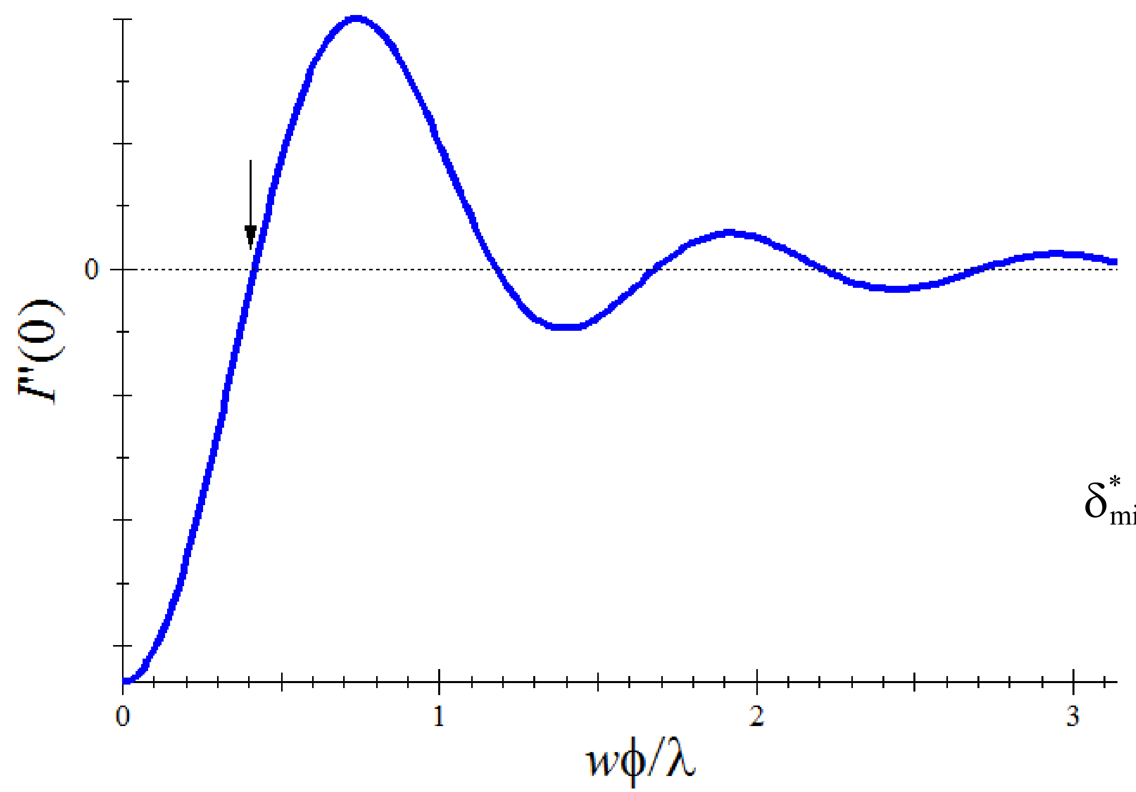


$$\Rightarrow \delta_{\min} = \frac{\lambda}{2\beta} = (0.5)\frac{\lambda}{\beta}$$

$$\textit{Note}: \text{ circular apertures} \Rightarrow \delta_{\min} = (0.61)\frac{\lambda}{\beta}$$

Modified Resolution Threshold for Slits

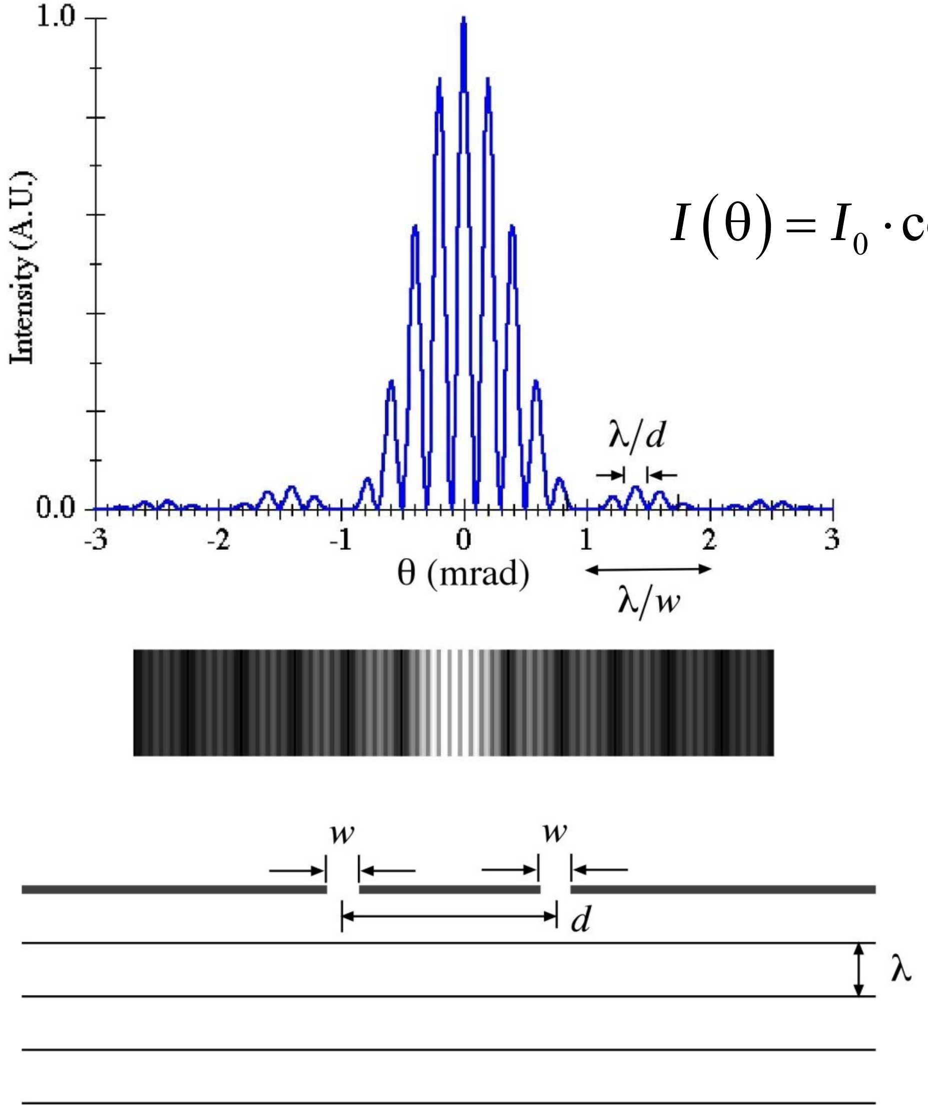
Just resolvable if central intensity is a local minimum



$$\phi^*_{min} = (0.41)\frac{\lambda}{w}$$

$$\delta^*_{min} = (0.41)\frac{\lambda}{\beta} \qquad \left(< \frac{\lambda}{2\beta} \right)$$

Two-Slit Interference with Diffraction



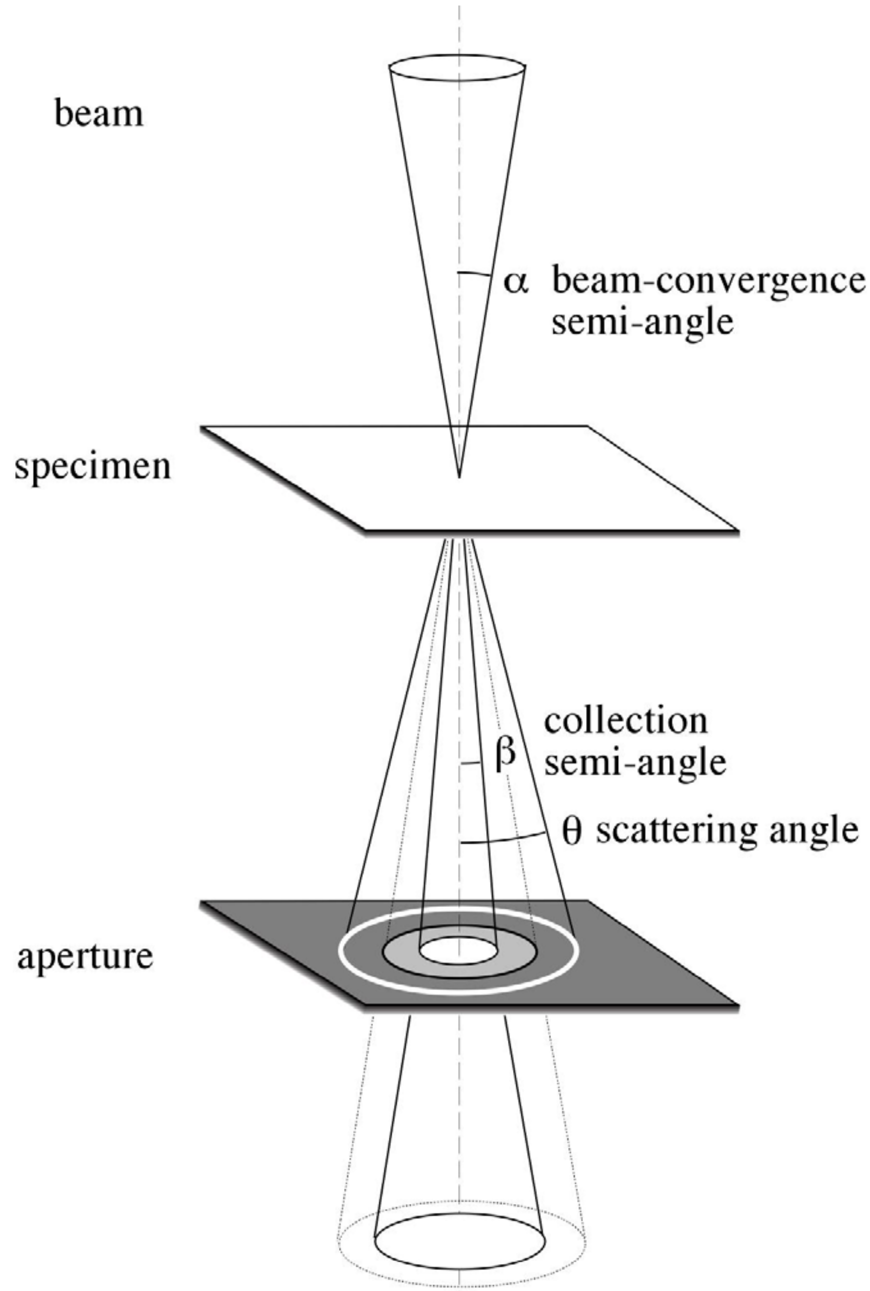
$$I(\theta) = I_0 \cdot \cos^2\left(\frac{\pi \cdot d \cdot \sin\theta}{\lambda}\right) \cdot \mathrm{sinc}^2\left(\frac{\pi \cdot w \cdot \sin\theta}{\lambda}\right)$$

Beam/Imaging Angles

Beam convergence used to
create small probe

Restricted collection angle to:
1) improve contrast
2) improve resolution

Some scattering outside of
collection angle



Electron Diffraction Patterns