5. Periodic Potentials

Problems

5.1. Please state the Bloch theorem for the wavefunction of an electron in a crystal. Does this imply that the electron wave function must have exactly the same periodicity as the crystal?

5.2. An electron is in a crystal with potential \( V(x) = 2V \cdot \cos(2\pi x / a) \), where \( V = 10 \text{ eV} \) and \( a = 2 \text{ nm} \). Based on a three-band analysis, find the bandgap of this crystal.

5.3. Plot the dispersion relation for a diatomic crystal for the following cases: a) \( m_1 = 2m_2 \), b) \( m_1 = 0.1m_2 \). Label the axes and branches. Assume any values of \( m_1 \) and \( k_1 \) and disregard the actual frequency values.