NANO 705
Homework 2
Due: F-Feb. 10, 10:00 AM

Show all work. Use additional sheets.

Consider the following Hamiltonian:

$$\hat{\mathrm{H}} = \frac{p^2}{2m} + V(x)$$

where

$$V(x) = \begin{cases} 0, & 0 \le x \le L \\ \infty, & x < 0 \text{ or } L < x \end{cases}$$

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1) Given the wave function:

$$\phi(x) = \begin{cases} \sqrt{\frac{2}{L}} \cdot \sin\left(\frac{\pi x}{L}\right), & 0 \le x < L \\ 0, & x < 0 \text{ or } L < x \end{cases}$$

a) Show that $\phi(x)$ is an eigenfunction of $\hat{H}$.

b) Show that $\phi(x)$ is normalized.

c) Find the energy E_0 of this state.

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2) Consider the wave function:

$$\psi(x) = \begin{cases} \sqrt{\frac{30}{L}} \cdot \left(\frac{x}{L}\right) \cdot \left(1 - \frac{x}{L}\right), & 0 \le x < L \\ 0, & x < 0 \text{ or } L < x \end{cases}$$

a) Show that $\psi(x)$ is not an eigenstate of $\hat{\mathrm{H}}$.

b) Show that $\psi(x)$ is normalized.

c) Find the expectation value $E = \langle \psi | \hat{\mathrm{H}} | \psi \rangle$ of the energy. Compare to the result in 1): Which is smaller?

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3) For $L = 10$ nm, $mc^2 = 511$ KeV, evaluate the energies (in eV) in 1) and 2). Plot the wave functions using Matlab.