NANO 705
Homework 3
Due: F-2/28, 10:00 AM

Show all work. Use additional sheets.

The spherical wave function

$$\phi(r) = \frac{1}{\sqrt{2\pi R}} \cdot \frac{e^{-r/R}}{r}$$

describes an electron in a bound state outside a very narrow, deep potential well, where $\hat{H} = \hat{K} = \hat{p}^2 / 2m$.

1) Find an expression for its kinetic energy K. [Hint: use $\hat{K}_r = -\left(\hbar^2/2m\right)\nabla_r{}^2$, where $\nabla_r{}^2 = \frac{1}{r}\frac{\partial^2}{\partial r^2} r$.]

2) Find an expression for the radial electron density $\sigma(r) = 4\pi r^2 \left|\phi(r)\right|^2$.

3) Assume other electrons are in the well and the electron-electron energy is

$$U_{ee}(r) = \frac{q^2}{6\pi\varepsilon_0 R} \cdot \left(\frac{a_0}{R}\right)^2 \cdot e^{-2r/R}$$

where

$$a_0 = \frac{4\pi\varepsilon_0\hbar^2}{mq^2} = 0.053 \text{ nm}$$

Find an expression for the expectation value of the U_{ee} using

$$U_{ee} = \int_{r=0}^{\infty} dr \cdot \sigma(r) \cdot U_{ee}(r)$$

4) Find the value $R = R_{best}$ that mimizes the total energy $E = K + U_{ee}$. Express R_{best} as a constant times a_0.

5) Find the minimum energy $E_{best} = E|_{R=R_{best}}$. Express E_{best} in rydbergs, where

$$1 \text{ Ryd} = \frac{\hbar^2}{2ma_0{}^2} = 13.6 \text{ eV}$$