A two-state system has the energy eigenfunctions and fermi function shown below:

state 1: $\Phi_1(x) = \sqrt{\dfrac{2}{L}} \cdot \cos(kx)$, $f_0(\varepsilon_1 - \mu) = \dfrac{1}{4}$

state 2: $\Phi_2(x) = \sqrt{\dfrac{2}{L}} \cdot \sin(kx)$, $f_0(\varepsilon_2 - \mu) = \dfrac{3}{4}$

1) Find the 1-D electron density $n(x)$:

2) Write down the equilbirium density matrix $[\rho]$:

3) Find the average number $\langle N \rangle$ of electrons in the system.

4) The representation of an operator $\hat{A}$ in this system is:

$[A] = \begin{pmatrix} -a & c \\ b & a \end{pmatrix}$

Find an expression for the expectation value $\langle \hat{A} \rangle$.