Introduction

Three generations of solar cells:

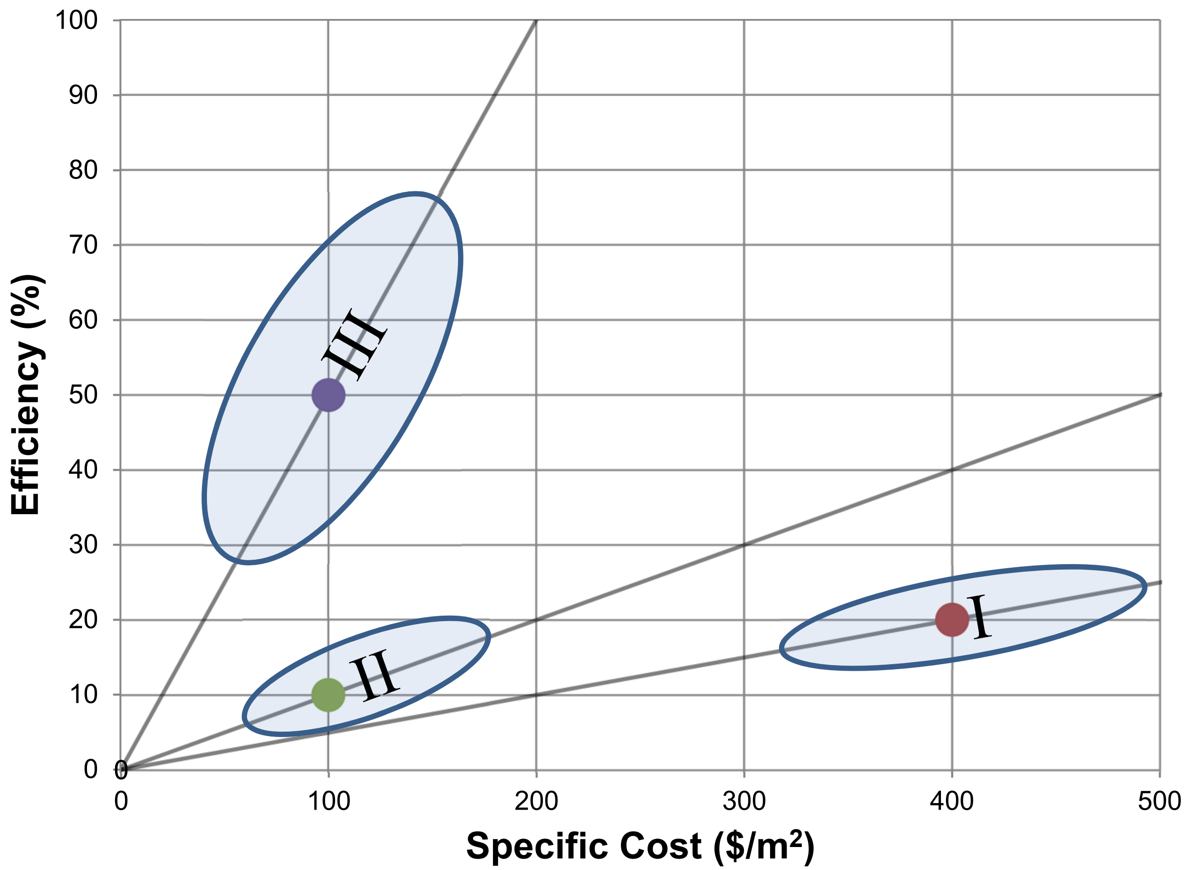
gen1: wafer-based (bulk) materials
•high materials specific cost (even despite abundant resources)
•moderate efficiency (15-25%)

gen2: amorphous and polycrystalline thin films
•lower materials demands reduces specific cost
•low-to moderate efficiency (8-15%)
•generally based on the same physics as gen1 photovoltaics (PV)

gen3: target subject of this course:
•maintain low specific cost
•improve efficiency
•requires new concepts in PV

Efficiency vs. Specific Cost

- Increase efficiency
- Reduce specific cost (price/area)



Another Benchmark

Relate efficiency/specific cost to $/Watt

η : efficiency (%)

C : cost ($)

A : area (m^2)

$\eta = P/P_S$

P : power generated (W)

P_S : solar power supplied (W)

$$\frac{\eta}{C/A} = \frac{P/P_S}{C/A} = \frac{P}{C\cdot(P_S/A)} = \frac{1}{\left(\frac{C}{P}\right)\cdot I_S}$$

I_S : solar power flux (W/m^2)

$$\frac{\eta}{C/A}\uparrow \Leftrightarrow \frac{C}{P}\downarrow$$

Decreasing cost/unit power is equivalent to increasing efficiency/specific cost.

Estimations

A common benchmark to remember was set by DOE, that is \$1/W.

$$\frac{C}{P} = \frac{1}{\left(\frac{\eta}{C/A}\right)\cdot I_S} = \frac{C/A}{\eta \cdot I_S} \qquad I_S \approx 1000 \ \mathrm{W/m^2}$$

gen 1:

$$\eta \approx 20\% \qquad \frac{C}{A} \approx \$400/m^2 \qquad \frac{C}{P} \approx \frac{\$400/m^2}{(0.20)\cdot\left(1000 \ W/m^2\right)} = \$2.00/W$$

gen 2:

$$\eta \approx 10\% \qquad \frac{C}{A} \approx \$100/\mathrm{m}^2 \qquad \frac{C}{P} \approx \frac{\$100/\mathrm{m}^2}{(0.10)\cdot\left(1000 \ \mathrm{W/m^2}\right)} = \$1.00/\mathrm{W}$$

gen 3: Can we do better?

→Need higher η,lower C/A

Approaches

What materials/structures are we talking about?

gen 1

- wafers of high-purity crystalline silicon (c-Si), GaAs, InP
- p/n homojunctions
- doping by diffusion or implantation
- high energy requirement for fabrication and materials demands

gen 2

- CdTe, $CuInSe_2$, a-Si:H, thin-film Si, CZTS (kesterites, stannites)
- evaporation, sputtering, dip coating on TCO coated glass
- heterojunctions with another material (CdS) or p/i/n structures
- scalable processes

gen 3

- semiconductor quantum dots, wires, wells, superlattices?
- rare-earth, up-converting nanoparticle composites?
- selective absorbers/emitters?
- nanoengineered photonic structures for light trapping?

Background

Photovoltaics: Directly convert light energy into electrical energy.

Light consists of photons (particles) with energy E, but can also be described as a wave , with frequency f and wavelength λ.

$$E = hf$$

Planck's constant

$$v = f \cdot \lambda_n = \frac{c}{n}$$

speed of light in vacuum

index of refraction

speed of light in medium

$$c = f \cdot \lambda_1$$ //frequency is constant

$$\lambda_n = \lambda_1 \cdot \left(\frac{v}{c}\right) = \frac{\lambda_1}{n}$$

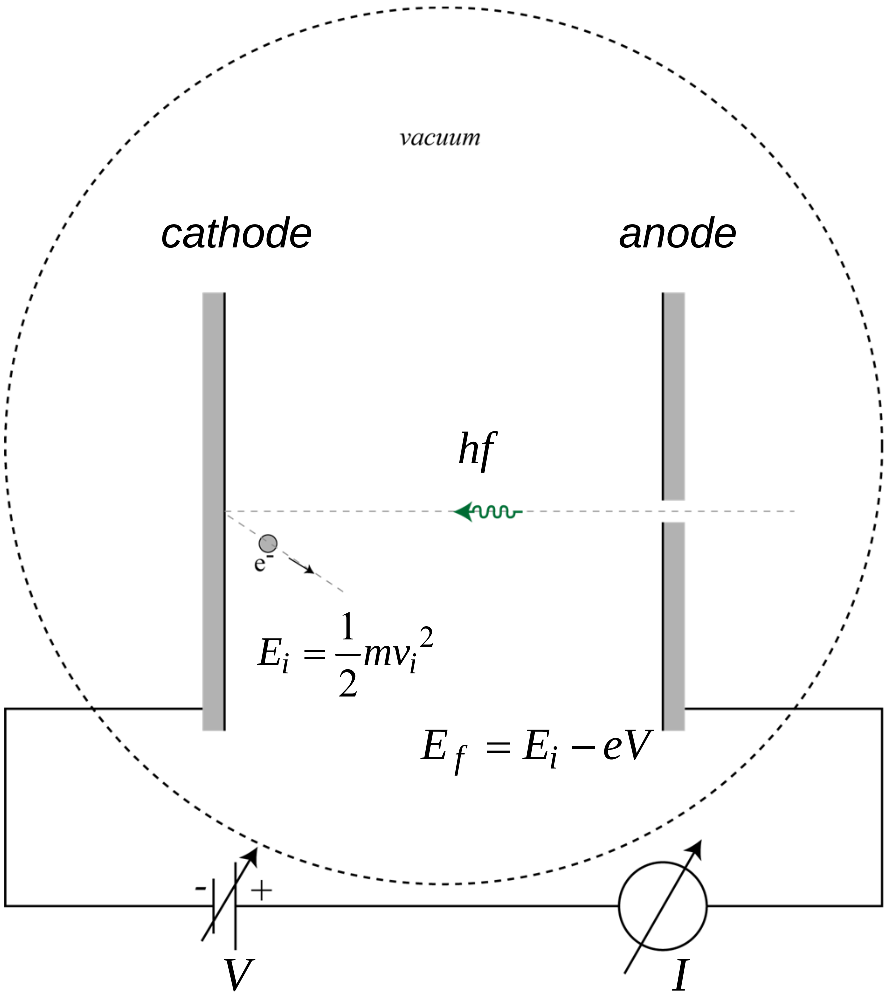
$$E = \frac{hv}{\lambda_n} = \frac{hc}{\lambda_1}$$

$$\left.\begin{aligned} h &= 6.63\times10^{-34}\ \text{J}\cdot\text{s} \\ c &= 3.00\times10^{8}\ \text{m/s} \\ 1\ \text{eV} &= 1.602\times10^{-19}\ \text{J} \end{aligned}\right\} \Rightarrow hc = 1240\ \text{eV}\cdot\text{nm}$$

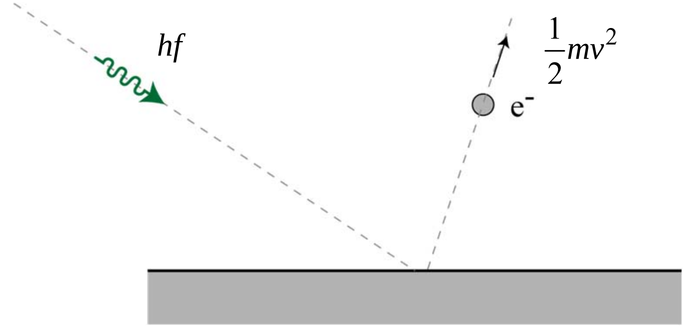
$$\lambda_1(visible) \approx 400-700\ \text{nm}$$

Light undergoes both collisions (particles) and interference (waves).

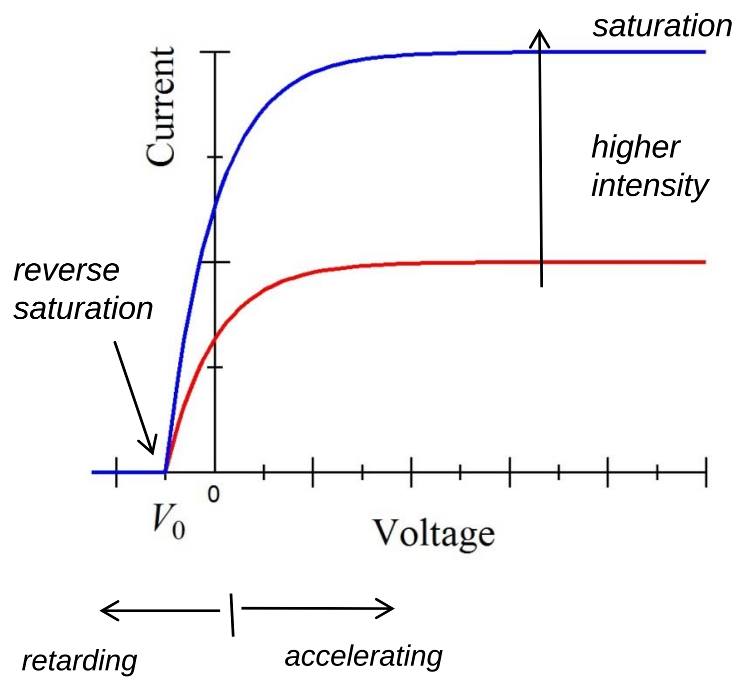
Photoelectric Effect (I)



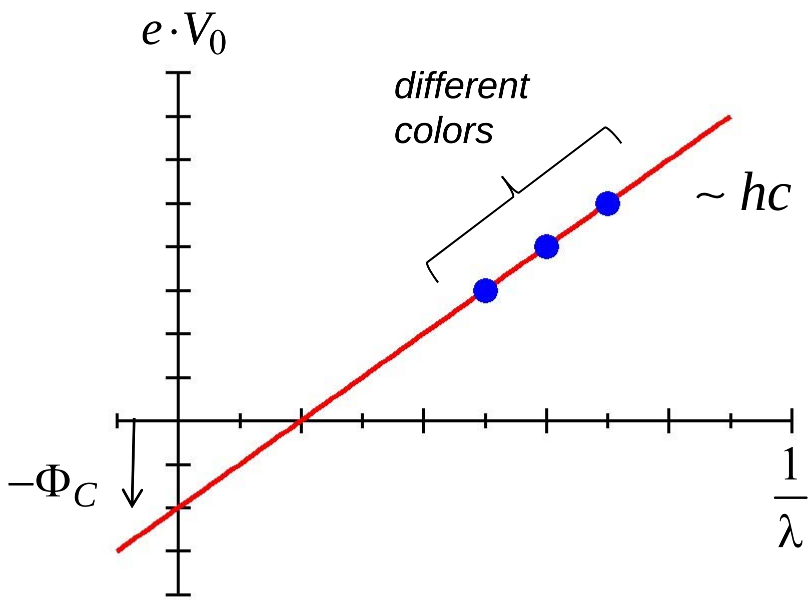
Φ_C: *cathode work function*



$$\frac{1}{2} m v_i^2 = hf - \Phi_C \qquad eV_0 = \frac{1}{2} m v_i^2$$



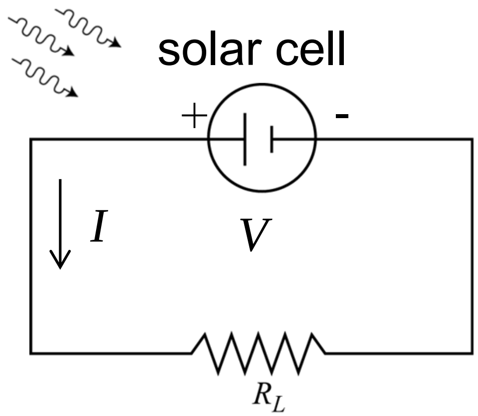
Photoelectric Effect (II)

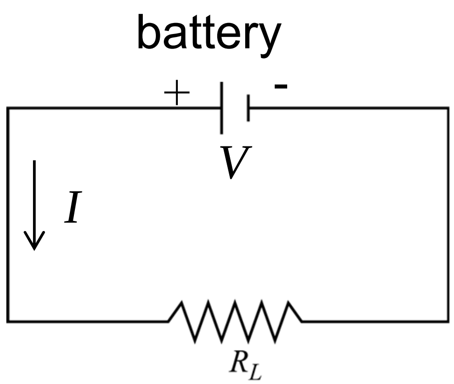


$$E_f = 0 \Rightarrow e \cdot V_0 = \frac{1}{2} m v_i^2 = hf - \Phi_C$$

$$eV_0 = \frac{hc}{\lambda} - \Phi_C$$

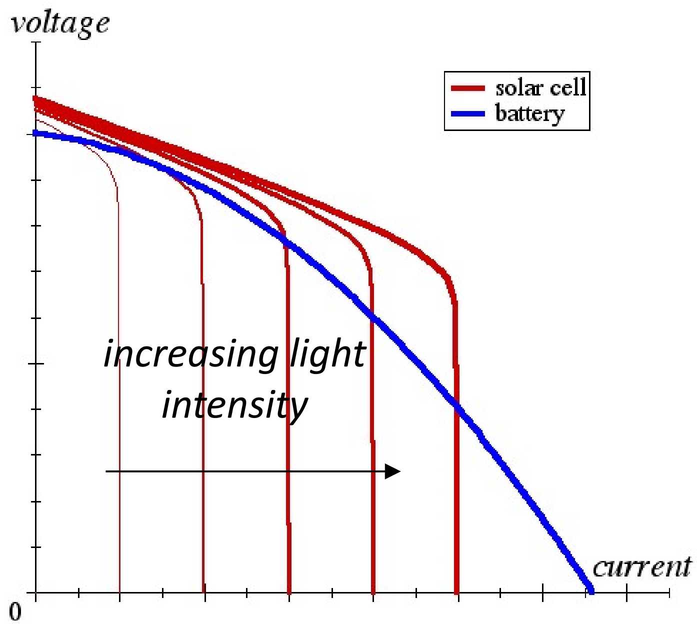
Solar Cell vs. Battery (Ideal)





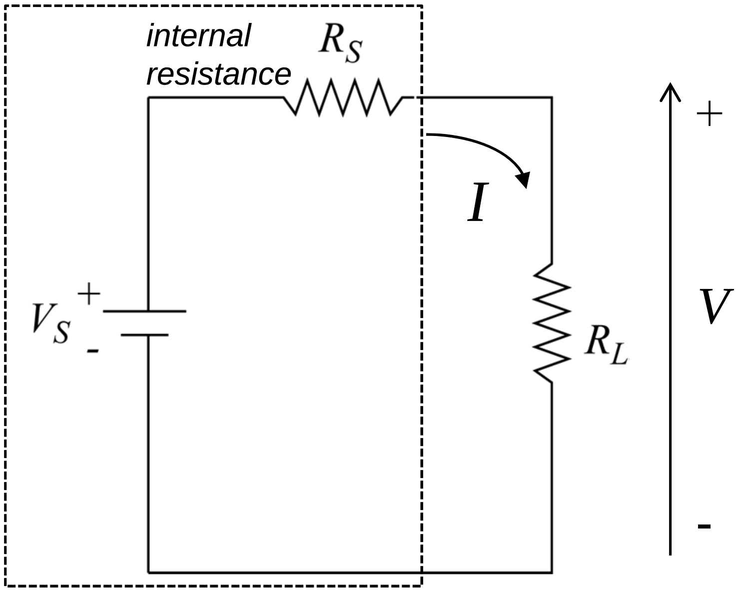
$$I(V) = I_{photo} - I_{dark}(V)$$

$$I_{dark}(V) = I_0 \cdot \left(e^{qV/nk_BT} - 1\right)$$



$$I(V) = \frac{V}{R_L}$$

Battery: equivalent circuit

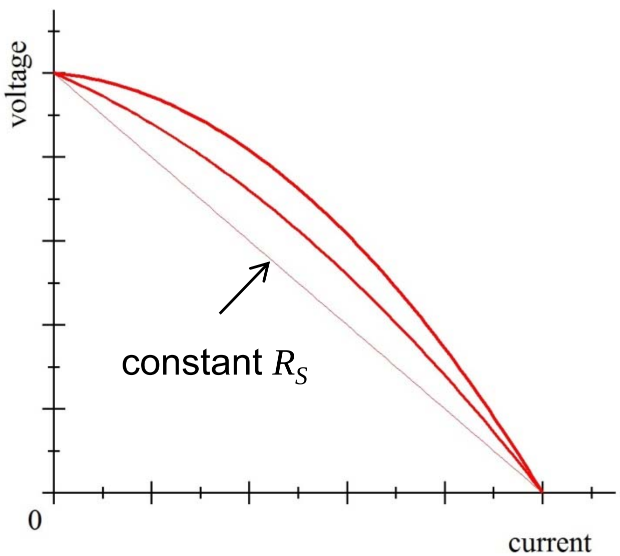


$$I = \frac{V_S}{R_S + R_L}$$

//in series

$$V = \left(\frac{R_L}{R_S + R_L}\right) \cdot V_S$$

//voltage divider



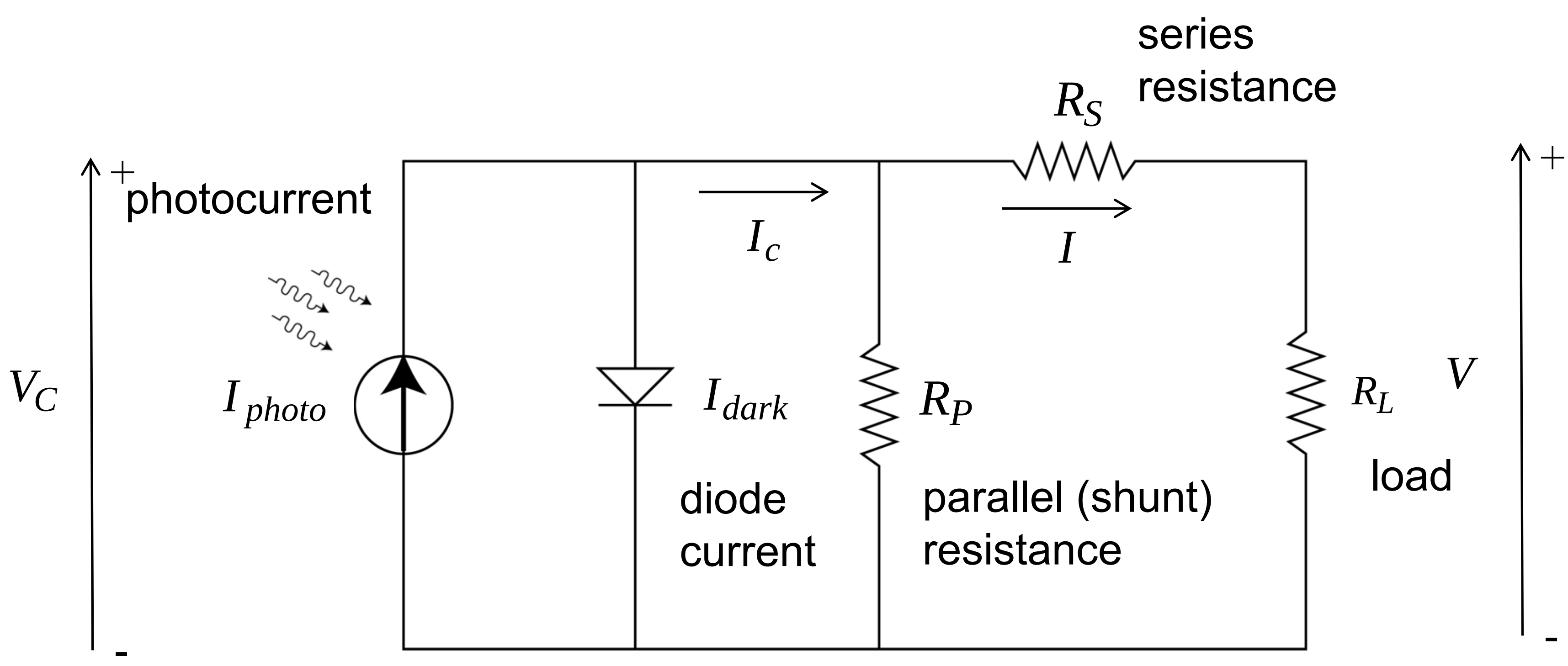
$$R_S = R_0 + R_1 \cdot \left(\frac{I}{I_{\max}}\right)$$

//assume internal resistance is current dependent

$$I_{\max} \equiv \frac{V_S}{R_0 + R_1}$$

$$V = V_S - I \cdot R_S$$

Solar cell: equivalent circuit

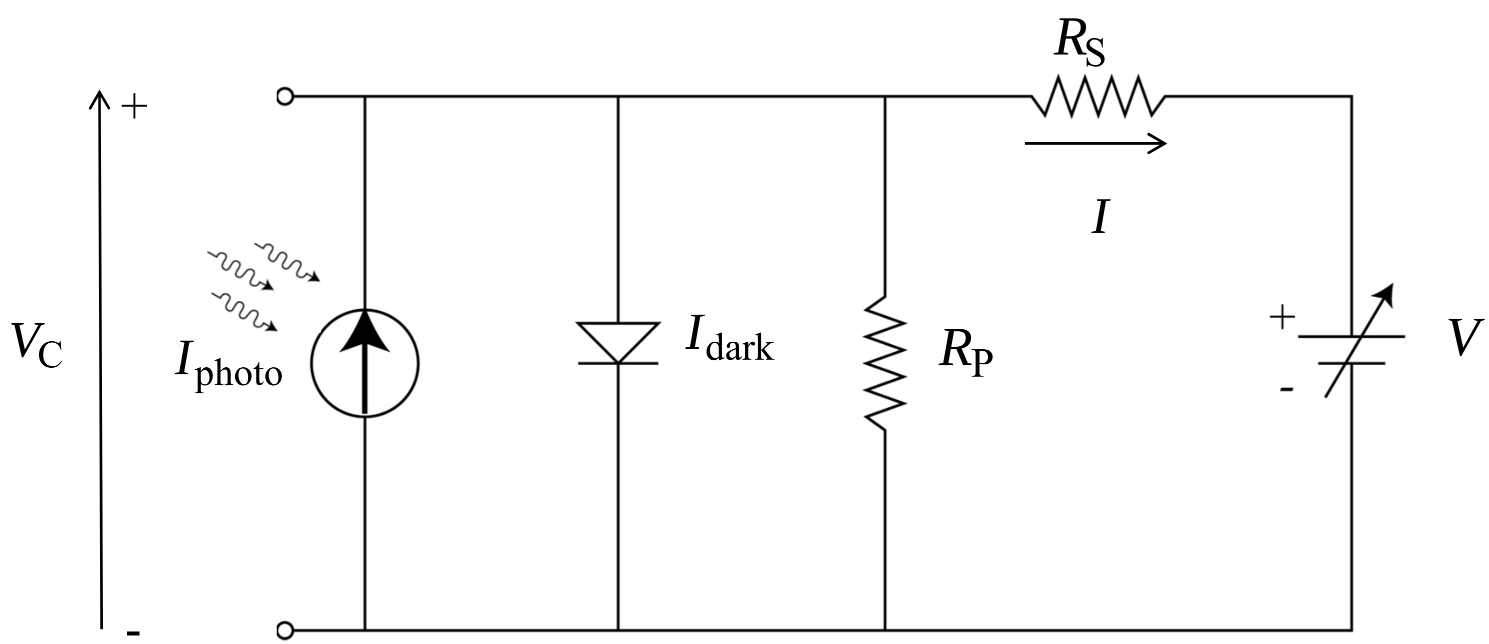


$$I_C(V_C) = I_{photo} - I_{dark}(V_C)$$

$$I = I_C(V_C) - \frac{V_C}{R_P} = \frac{V_C - V}{R_S}$$

$$V_c = I \cdot (R_S + R_L) \qquad V = I \cdot R_L$$

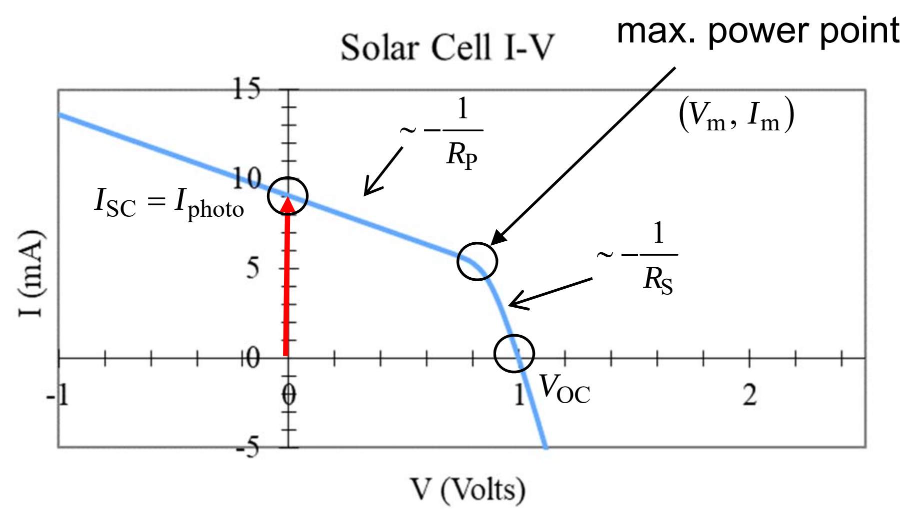
Solar-cell testing



$$I_C(V_C) = I_{photo} - I_{dark}(V_C)$$

$$V = V_C - I \cdot R_S$$

$$I = I_C(V_C) - \frac{V_C}{R_P} = \frac{V_C - V}{R_S}$$



Parasitic impedances (I)

$$I = I_C - \frac{V_C}{R_P}$$

$$V = \left(1 + \frac{R_S}{R_P}\right) \cdot V_C - I_C \cdot R_S$$

$$\frac{\partial I}{\partial V_C} = \frac{\partial I_C}{\partial V_C} - \frac{1}{R_P}$$

$$R_P \gg R_S$$

$$\frac{\partial V}{\partial V_C} = 1 + \frac{R_S}{R_P} - R_S \frac{\partial I_C}{\partial V_C}$$

$$\frac{\partial I}{\partial V} \approx \frac{1}{R_S} \cdot \left(\frac{\frac{\partial I_C}{\partial V_C} - \frac{1}{R_P}}{\frac{1}{R_S} - \frac{\partial I_C}{\partial V_C}} \right)$$

$$\frac{\partial I_C}{\partial V_C} = \frac{1}{nR_0} \cdot e^{qV_C/nkT}$$

$$R_0 = \frac{kT}{qI_0}$$ //big

Short-circuit:

$$V_C \approx 0$$

$$\frac{\partial I_C}{\partial V_C} \ll \frac{1}{R_P} \ll \frac{1}{R_S} \quad \Longrightarrow \quad \frac{\partial I}{\partial V} = -\frac{1}{R_P}$$

Open-circuit:

$$V_C \approx V_{OC}$$

$$\frac{1}{R_P} \ll \frac{1}{R_S} \ll \frac{\partial I_C}{\partial V_C} \quad \Longrightarrow \quad \frac{\partial I}{\partial V} = -\frac{1}{R_S}$$

Parasitic impedances (II)

Short-circuit:

$$V_C \approx 0$$

$$I_C = I_{ph} + \cancel{I_{dark}(V_C)}$$

$$I_{ph} - \frac{V_C}{R_P} \approx \frac{V_C - V}{R_S}$$

$$\left(\cancel{\frac{1}{R_P}} + \frac{1}{R_S} \right) \cdot V_C = I_{ph} + \frac{V}{R_S}$$

$$V_C = I_{ph} \cdot R_S + V$$

$$I = I_{ph} - \frac{I_{ph} \cdot R_S + V}{R_P} = \left(1 - \cancel{\frac{R_S}{R_P}} \right) \cdot I_{ph} - \frac{V}{R_P}$$

$$\frac{\partial I}{\partial V} = -\frac{1}{R_P}$$

Open-circuit:

$$I = I_C - \frac{V_C}{R_P} \approx 0$$

$$V_C = I_C \cdot R_P$$

$$I = \frac{V_C - V}{R_S} \approx \frac{I_C \cdot R_P - V}{R_S}$$

$$\frac{\partial I}{\partial V} = -\frac{1}{R_S}$$

I-V Algorithm

//Initialize

$I = 0$ $V_C = V$

$tol = 10^{-12}$ *done*=false

$$f_V(V_C) \equiv V_C \cdot \left(1 + \frac{R_S}{R_P}\right) - I_C(V_C) \cdot R_S$$

while(not(done))

$improved = \text{false}$

{ $V'_+ = f_V(V_C + \delta V)$

if $\left(|V'_+ - V| < \delta_{best}\right)$

{

$\delta_{best} = |V'_+ - V|$

$V_C \to V_C + \delta V$

$improved = \text{true}$

}

$V'_- = f_V(V_C - \delta V)$

if $\left(|V'_- - V| < \delta_{best}\right)$

{

$\delta_{best} = |V'_- - V|$

$V_C \to V_C - \delta V$

$improved = \text{true}$

}

if(improved) $\delta V \to (1.5) \cdot \delta V$

else $\delta V \to \delta V / 2$

if $\left(\delta_{best} < tol\right)$ *done*=true

}

Dist. Function (I)

Find Bose-Einstein and Fermi-Dirac distribution functions

$\Omega(E,N)$ //# of states w/ energy E and N particles

$S = k\cdot\ln\left[\Omega(E,N)\right]$ //entropy

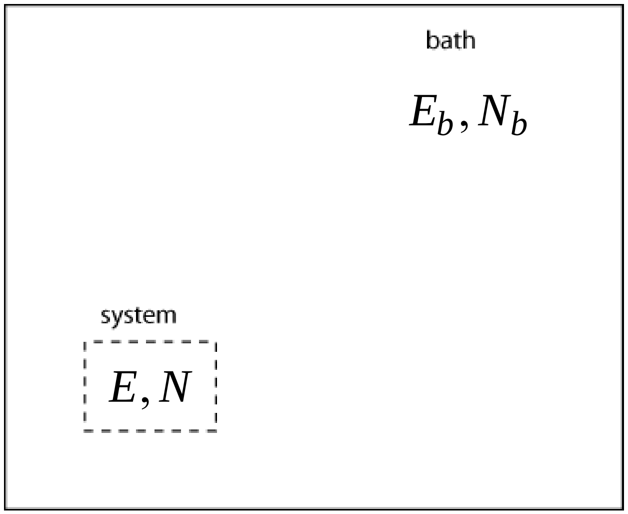
$dE = T\cdot dS + \mu\cdot dN$ //change in internal energy w/ changes in S and N

temperature (T) chemical potential (μ)

$$\frac{1}{kT} = \frac{1}{k}\cdot\left(\frac{\partial S}{\partial E}\right)_N = \frac{\partial}{\partial E'}\left\{\ln\left[\Omega(E',N')\right]\right\}\Big|_{\substack{E'=E\\N'=N}}$$

$$\frac{-\mu}{kT} = \frac{1}{k}\cdot\left(\frac{\partial S}{\partial N}\right)_E = \frac{\partial}{\partial N'}\left\{\ln\left[\Omega(E',N')\right]\right\}\Big|_{\substack{E'=E\\N'=N}}$$

Dist. Function (II)



$$E_{tot} = E_b + E$$
$$E \ll E_{tot}$$
$$E_b \approx E_{tot}$$

$$N_{tot} = N_b + N$$
$$N \ll N_{tot}$$
$$N_b \approx N_{tot}$$

$$\begin{aligned}\Omega_{tot}\left(E_{tot}, N_{tot}\right) &= \Omega_b\left(E_b, N_b\right)\cdot\Omega\left(E, N\right)\\ &= \Omega_{tot}\left(E_{tot} - E, N_{tot} - N\right)\cdot\Omega\left(E, N\right)\end{aligned}$$

Note:

$$\begin{aligned}S_{tot} &= k\ln\left[\Omega_{tot}\left(E_{tot}, N_{tot}\right)\right]\\ &= k\ln\left[\Omega_b\left(E_b, N_b\right)\right] + k\ln\left[\Omega\left(E, N\right)\right]\end{aligned}$$

$$S_{tot} = S_b + S$$

//entropy is extensive

Dist. Function (III)

From calculus:

$$f\left(x'-\delta_x, y'-\delta_y\right)_{\substack{x'=x\\y'=y}} \approx \left[f\left(x',y'\right) - \delta_x \cdot \frac{\partial f\left(x',y'\right)}{\partial x'} - \delta_y \cdot \frac{\partial f\left(x',y'\right)}{\partial y'} \right]_{\substack{x'=x\\y'=y}}$$

$$\ln\left[\Omega_b\left(E'-E, N'-N\right)\right]\Big|_{\substack{E'=E_{tot}\\N'=N_{tot}}}$$

$$\approx \left\{ \ln\left[\Omega_b\left(E',N'\right)\right] - E \cdot \frac{\partial}{\partial E'} \ln\left[\Omega_b\left(E',N'\right)\right] - N \cdot \frac{\partial}{\partial N'} \ln\left[\Omega_b\left(E',N'\right)\right] \right\}\Big|_{\substack{E'=E_{tot}\\N'=N_{tot}}}$$

So:

$$\ln\left[\Omega_b\left(E_{tot}-E, N_{tot}-N\right)\right] \approx \ln\left[\Omega_b\left(E_{tot}, N_{tot}\right)\right] - \frac{\left(E-N\mu\right)}{kT}$$

$$P\left(E,N\right) \propto \Omega_b\left(E_{tot}-E, N_{tot}-N\right) = \Omega_b\left(E_{tot}, N_{tot}\right) \cdot \mathrm{e}^{-\frac{\left(E-N\mu\right)}{kT}}$$

//Probability that system has energy E, N particles

Dist. Function (IV)

Say the system is a single state with N_ℓ particles, each with energy E

$$E_\ell = N_\ell \cdot E$$

The probability that the state has N particles is:

$$P(N) = \frac{\mathrm{e}^{-N\cdot(E-\mu)/kT}}{\sum_n \mathrm{e}^{-n\cdot(E-\mu)/kT}}$$

The average # of particles in the state is:

$$\langle N \rangle = \sum_n n \cdot P(n) = f_\mu(E)$$

Define: $x \equiv \mathrm{e}^{-(E-\mu)/kT}$

$$f_\mu(E) = f(x)$$

$$f(x) = \frac{\sum_n n \cdot x^n}{\sum_n x^n}$$

We have two cases:

i) Bose-Einstein particles (bosons)
e.g., photons

$$n = 0, 1, 2, 3, \ldots \infty$$

ii) Fermi-Dirac particles (fermions)
e.g., electrons

$$n = 0, 1$$

Dist. Function (V)

case i) bosons

$$f(x) = \frac{\sum_{n=0}^{\infty} n \cdot x^n}{\sum_{n=0}^{\infty} x^n}$$

define: $S = \sum_{n=0}^{\infty} x^n$

$$-\left(xS = \sum_{n=1}^{\infty} x^n\right)$$

$$(1-x) \cdot S = 1 \Rightarrow S = \frac{1}{1-x}$$

Notice: $\sum_{n=0}^{\infty} n \cdot x^n = x \cdot \sum_{n=0}^{\infty} n \cdot x^{n-1} = x \cdot \frac{dS}{dx} = \frac{x}{(1-x)^2}$

$$f(x) = \frac{x}{1-x} = \frac{1}{\frac{1}{x} - 1}$$

$$f_\mu^{(\mathrm{BE})}(E) = \frac{1}{\mathrm{e}^{(E-\mu)/kT} - 1}$$

//Bose-Einstein distribution function

case ii) fermions

$$f(x) = \frac{\sum_{n=0}^{1} n \cdot x^n}{\sum_{n=0}^{1} x^n} = \frac{x}{1+x} = \frac{1}{\frac{1}{x} + 1}$$

$$f_\mu^{(\mathrm{FD})}(E) = \frac{1}{\mathrm{e}^{(E-\mu)/kT} + 1}$$

//Fermi-Dirac distribution function

limit $\varepsilon - \mu \gg kT$

$$f_\mu^{(\mathrm{MB})}(E) = \mathrm{e}^{-(E-\mu)/kT}$$

//Maxwell-Boltzmann distribution function