Drift current

The Drude model describes conduction by random collision events between carriers and fixed atoms

$$F = -q\mathrm{E} = ma \qquad a = -q\mathrm{E}/m \qquad \langle v \rangle = a\tau = -q\tau\mathrm{E}/m$$

force *mass* *acceleration*

average "drift" velocity *"relaxation time"*

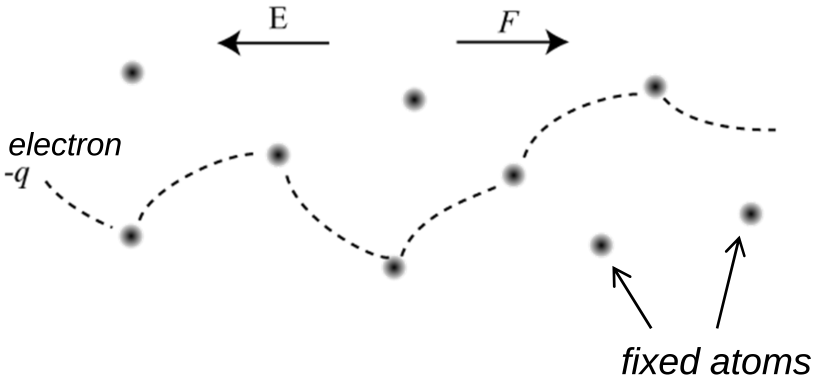
$$\langle J \rangle = -qn\langle v \rangle = \frac{q^2 n\tau}{m}\cdot\mathrm{E}$$

average current density

$$\langle v \rangle = -\mu\mathrm{E}$$

mobility

$$\mu = \frac{q\tau}{m} \qquad [\mu] = \frac{\mathrm{cm}^2}{\mathrm{V}\cdot\mathrm{s}}$$



Mobility is proportionality between electric field and "drift" velocity.

Ambipolar drift:

$$J_{\mathrm{n}}^{(\mathrm{drift})} = q\cdot\mu_n\cdot n\cdot\mathbf{E}$$

$$J_p^{(\mathrm{drift})} = q\cdot\mu_p\cdot p\cdot\mathbf{E}$$

conductivity

$$J^{(\mathrm{drift})} = J_n^{(\mathrm{drift})} + J_p^{(\mathrm{drift})} = \sigma\mathbf{E}$$

$$\sigma = q\cdot\left(\mu_n\cdot n + \mu_p\cdot p\right)$$

Diffusion current

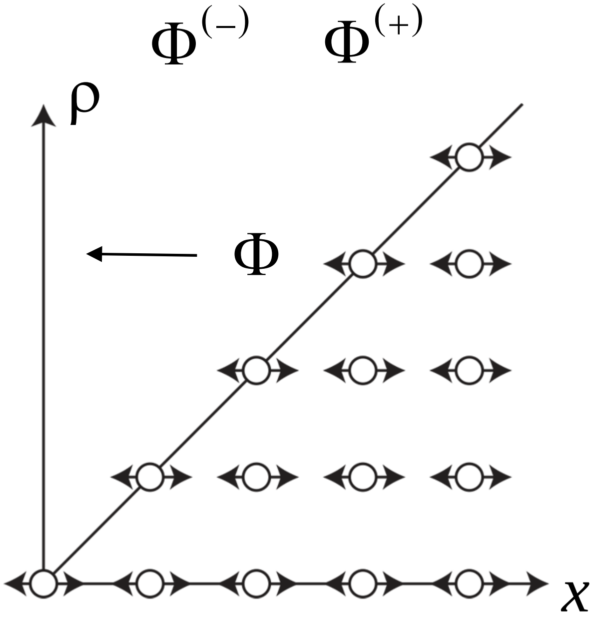
Fick's Law describes diffusion from concentration gradients.

$$\overset{\textit{flux}}{\Phi} = -\overset{\textit{diffusion constant}}{D} \cdot \overbrace{\frac{d\rho}{dx}}^{\textit{concentration gradient}}$$

$$[\Phi] = \frac{\#}{\mathrm{cm}^2 \cdot \mathrm{s}} \qquad [D] = \frac{\mathrm{cm}^2}{\mathrm{s}} \qquad \left[\frac{d\rho}{dx}\right] = \frac{\#}{\mathrm{cm}^4}$$

$$J_n^{(\mathrm{diff})} = -q \cdot \Phi_n = -q \cdot \left(-D_n \cdot \frac{dn}{dx}\right) = qD_n \cdot \frac{dn}{dx}$$

$$J_p^{(\mathrm{diff})} = +q \cdot \Phi_p = q \cdot \left(-D_p \cdot \frac{dp}{dx}\right) = -qD_p \cdot \frac{dp}{dx}$$



Einstein relations

$$J_n = q\overset{\textit{mobility}\ \searrow}{\mu_n} n\mathrm{E} + q\overset{\swarrow\ \textit{diffusion constant}}{D_n}\cdot\frac{dn}{dx}$$ //electron current density

$$J_p = \underbrace{q\mu_p p\mathrm{E}}_{\textit{drift}} - \underbrace{qD_p\cdot\frac{dp}{dx}}_{\textit{diffusion}}$$ //hole current density

The E-field represents a change in potential energy w.r.t. position: $\mathrm{E} = -\frac{dV}{dx}$

$$E_C(x) = E_{C0} - qV(x)$$ //potential energy of CB electron

$$n(x) = N_C\cdot e^{-[E_C(x)-E_F]/kT}$$ //electron concentration

$$\frac{dn}{dx} = -\left(\frac{1}{kT}\right)\cdot\frac{dE_C}{dx}\cdot n(x) = \frac{-q\mathrm{E}}{kT}\cdot n(x)$$

In equilibrium: $J_n = 0 = \cancel{q}\mu_n\cancel{n}\,\cancel{\mathrm{E}} - \cancel{q}D_n\cdot\left(\frac{q\cancel{\mathrm{E}}}{kT}\right)\cdot\cancel{n}$

$$\frac{D_n}{\mu_n} = \frac{kT}{q} \quad \textit{and} \quad \frac{D_p}{\mu_p} = \frac{kT}{q}$$ //Einstein relations

Continuity equation (I)

For electrons:

generation rate

recombination rate

$$\frac{\partial n}{\partial t} = \frac{1}{q}\frac{\partial J_n}{\partial x} + G_n - U_n$$

Steady state: $\frac{\partial n}{\partial t} = 0$ → $0 = \frac{1}{q}\frac{dJ_n}{dx} + G_n - U_n$

For holes:

$$\frac{\partial p}{\partial t} = -\frac{1}{q}\frac{\partial J_p}{\partial x} + G_p - U_p$$

Steady state: $\frac{\partial p}{\partial t} = 0$ → $0 = -\frac{1}{q}\frac{dJ_p}{dx} + G_p - U_p$

Continuity Equation (II)

For electrons:

$$J_n = q\mu_n n\mathrm{E} + qD_n \cdot \frac{dn}{dx}$$

Steady state: $$-\frac{1}{q}\frac{dJ_n}{dx} = G_n - U_n$$

$$\frac{dJ_n}{dx} = q\mu_n \cdot \frac{dn}{dx} \cdot \mathrm{E} + q\mu_n \cdot n \cdot \frac{d\mathrm{E}}{dx} + qD_n \cdot \frac{d^2n}{dx^2}$$

$$\longrightarrow \quad -D_n \cdot \frac{d^2n}{dx^2} - \mu_n \cdot \mathrm{E} \cdot \frac{dn}{dx} - \mu_n \cdot \frac{d\mathrm{E}}{dx} \cdot n = G_n - U_n$$

For holes:

$$J_p = q\mu_p p\mathrm{E} - qD_p \cdot \frac{dp}{dx}$$

Steady state: $$\frac{1}{q}\frac{dJ_p}{dx} = G_p - U_p$$

$$\frac{dJ_p}{dx} = q\mu_p \cdot \frac{dp}{dx} \cdot \mathrm{E} + q\mu_p \cdot p \cdot \frac{d\mathrm{E}}{dx} - qD_p \cdot \frac{d^2p}{dx^2}$$

$$\longrightarrow \quad -D_p \cdot \frac{d^2p}{dx^2} + \mu_p \cdot \mathrm{E} \cdot \frac{dp}{dx} + \mu_p \cdot \frac{d\mathrm{E}}{dx} \cdot p = G_p - U_p$$

Non-Equilibrium, Steady State

-illumination
-applied bias, constant current

$$\begin{tikzcd} & f_n(E,T_n) \\ f \arrow[ur] \arrow[dr] & \\ & f_p(E,T_p) \end{tikzcd}$$

quasi-equilibrium

$$f_n = \frac{1}{e^{(E-E_{\mathrm{F}n})/kT_n}+1} \rightarrow \frac{1}{e^{(E-E_{\mathrm{F}n})/kT}+1}$$

no “hot” carriers: $T_n \approx T_p = T$

$$f_p = \frac{1}{e^{(E-E_{\mathrm{F}p})/kT_p}+1} \rightarrow \frac{1}{e^{(E-E_{\mathrm{Fp}})/kT}+1}$$

separate equilibrium distributions within CB and VB

$E_{\mathrm{F}_n}, E_{\mathrm{F}_p}$: electron/hole quasi-Fermi levels

$$n = n_i \cdot e^{(E_{\mathrm{F}n}-E_i)/kT} = N_{\mathrm{C}} \cdot e^{(E_{\mathrm{C}}-E_{\mathrm{F}n})/kT}$$

$$p = n_i \cdot e^{(E_i-E_{\mathrm{F}p})/kT} = N_{\mathrm{V}} \cdot e^{(E_{\mathrm{F}p}-E_{\mathrm{V}})/kT}$$

$$n \cdot p = n_i^2 \cdot e^{(E_{\mathrm{F}n}-E_{\mathrm{F}p})/kT} = \underbrace{n_i^2 \cdot e^{\Delta\mu/kT}}_{\text{“law of mass action”}}$$

$\Delta\mu$: Quasi-Fermi level splitting

Steady-state current

In steady-state, near equilibrium, we assume:

$$\left.\begin{aligned} D_n &= \frac{kT}{q}\cdot \mu_n \\ D_p &= \frac{kT}{q}\cdot \mu_p \end{aligned}\right] \rightarrow \left[\begin{aligned} J_n &= \mu_n \cdot \left(qn\mathrm{E} + kT \cdot \frac{dn}{dx} \right) \\ J_p &= \mu_p \cdot \left(qp\mathrm{E} - kT \cdot \frac{dp}{dx} \right) \end{aligned}\right.$$

In quasi-equilibrium, steady-state:

$$\left.\begin{aligned} n(x) &= n_i \cdot \mathrm{e}^{\left[E_{\mathrm{F}_n}(x)-E_i(x)\right]/kT} \\ p(x) &= n_i \cdot \mathrm{e}^{\left[E_i(x)-E_{\mathrm{F}_p}(x)\right]/kT} \end{aligned}\right]$$

Use:

$$\frac{dE_i}{dx} = q\mathrm{E} \rightarrow \left[\begin{aligned} kT\cdot\frac{dn}{dx} &= \left(\frac{dE_{\mathrm{F}_n}}{dx} - \frac{dE_i}{dx}\right)\cdot n = \left(\frac{dE_{\mathrm{F}_n}}{dx} - q\mathrm{E}\right)\cdot n \\ kT\cdot\frac{dp}{dx} &= \left(\frac{dE_i}{dx} - \frac{dE_{\mathrm{F}_p}}{dx}\right)\cdot p = \left(q\mathrm{E} - \frac{dE_{\mathrm{F}_p}}{dx}\right)\cdot p \end{aligned}\right.$$

$$J_n = \mu_n \cdot n \cdot \frac{dE_{\mathrm{F}_n}}{dx}$$

$$J_p = \mu_p \cdot p \cdot \frac{dE_{\mathrm{F}_p}}{dx}$$

Currents are related to the slopes of the quasi-Fermi levels.

Diffusion length

$n = n_0 + \Delta n$ ← excess carrier conc.

$p = p_0 + \Delta p$

$n_0 \cdot p_0 = n_i^{\,2}$

Recombination:

$$U_n = \frac{dn}{dt} = \frac{d\left(\Delta n\right)}{dt} = \frac{\Delta n}{\tau_n} = \frac{n - n_0}{\tau_n} \qquad U_p = \frac{dp}{dt} = \frac{d\left(\Delta p\right)}{dt} = \frac{\Delta p}{\tau_p} = \frac{p - p_0}{\tau_p}$$

Steady state: $\frac{\partial n}{\partial t} = 0 \quad \rightarrow \quad 0 = \frac{1}{q}\frac{dJ_n}{dx} + G_n - U_n$

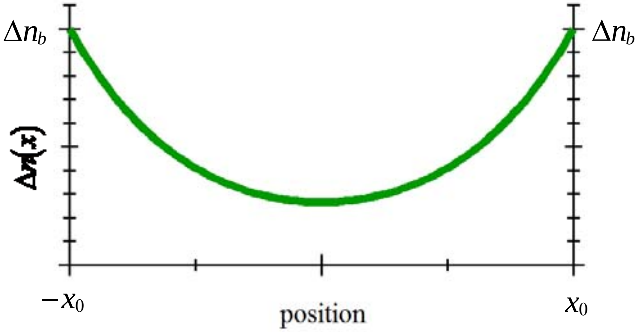
$$0 = D_n \cdot \frac{d^2 n}{dx^2} + \mu_n \cdot \mathrm{E} \cdot \frac{dn}{dx} + \mu_n \cdot \frac{d\mathrm{E}}{dx} \cdot n + G_n - U_n$$

Assume: $\mathrm{E} = 0 \qquad \frac{d\mathrm{E}}{dx} = 0 \qquad G_n = 0 \qquad U_n = \frac{n - n_0}{\tau_n}$

electron diffusion length

Define: $L_n = \sqrt{D_n \cdot \tau_n} \quad \rightarrow \quad \frac{d^2 n}{dx^2} = \frac{n - n_0}{L_n^{\,2}}$

Diffusion problem



Define: $\Delta n = n - n_0$

$$\rightarrow \quad \frac{d^2(\Delta n)}{dx^2} = \frac{\Delta n}{{L_n}^2}$$

Form of solution: $\Delta n(x) = A \cdot \mathrm{e}^{x/L_n} + B \cdot \mathrm{e}^{-x/L_n} = (A+B)\cdot\cosh\left(\frac{x}{L_n}\right) + (A-B)\cdot\sinh\left(\frac{x}{L_n}\right)$

We typically solve using boundary conditions:

Example: $\Delta n\big|_{x=-x_0} = \Delta n\big|_{x=x_0} = \Delta n_b$

$$\Delta n_b = (A+B)\cdot\cosh\left(\frac{x_0}{L_n}\right) - (A-B)\cdot\sinh\left(\frac{x_0}{L_n}\right)$$

$$= (A+B)\cdot\cosh\left(\frac{x_0}{L_n}\right) + (A-B)\cdot\sinh\left(\frac{x_0}{L_n}\right)$$

$$A = B$$

$$A + B = \frac{\Delta n_b}{\cosh\left(\frac{x_0}{L_n}\right)}$$

$$\rightarrow \qquad \Delta n(x) = \Delta n_b \cdot \frac{\cosh\left(\frac{x}{L_n}\right)}{\cosh\left(\frac{x_0}{L_n}\right)}$$

$$\Delta n(0) = \frac{\Delta n_b}{\cosh\left(\frac{x_0}{L_n}\right)}$$