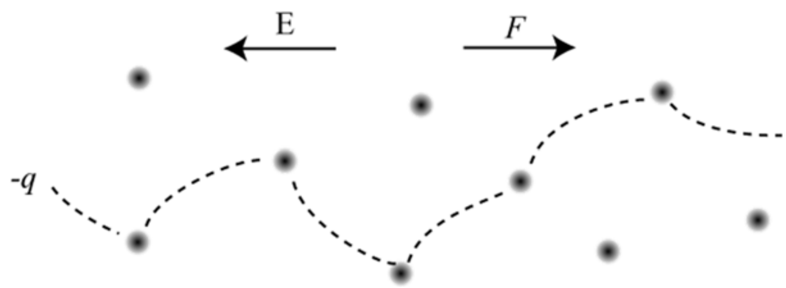
4. Transport

Drift current

The Drude model describes conduction by random collision events between carriers and fixed atoms. A uniform electric field accelerates the charge:

$$F = \pm qE = ma$$



The mean current density is proportional to the mean velocity

$$\langle J \rangle = \pm qn \langle v \rangle = \frac{q^2 n E}{m} \cdot \tau = \sigma E$$

using the relaxation time to relate a and <v>. $\langle v \rangle = a \cdot \tau$. The conductivity is:

$$\sigma = \frac{q^2 n \tau}{m} = q \mu n$$

which refers to the carrier mobility:

$$\mu = \frac{q\tau}{m}$$

Mobility is the proportional of the mean carrier velocity to electric field, called drift. The drift velocity is:

$$\langle v \rangle = \pm \mu E$$

with units $[\mu] = \text{cm}^2/\text{V} \cdot \text{s}$. So

$$\langle J \rangle = qn\mu E$$

Combining both carrier types present

$$J = J_n + J_p = q \cdot (\mu_n \cdot n + \mu_p \cdot p) \cdot E = \sigma E$$

and

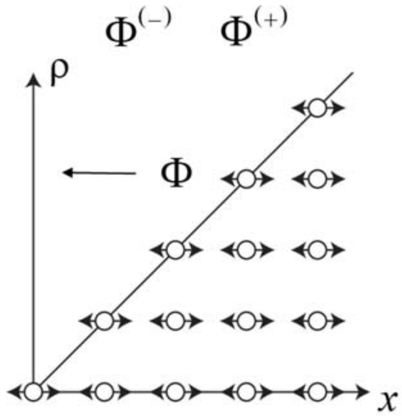
$$\sigma = q \cdot (\mu_n \cdot n + \mu_p \cdot p)$$

Diffusion current

Electron and hole transport of charge must also include the many-body effects of diffusion, which results from concentration gradients. Fick's law assumes that the flux Φ of diffusing particles (with units $[\Phi] = \#/(\text{cm}^2 \cdot \text{s})$) is proportional to the concentration gradient

$$\Phi = -D \cdot \frac{d\rho}{dx}$$

where D is the diffusion constant (with units $[D] = \mathrm{cm}^2/\mathrm{s}$) and $d\rho/dx$ is the slope in particle concentration ρ. Essentially, random motions to the right and left lead to a net flux in the direction of lower concentration.



For electrons and holes, this gives the diffusion currents

$$J_n^{(\mathrm{diff})} = -q \cdot \Phi_n = -q \cdot \left(-D_n \cdot \frac{dn}{dx}\right) = qD_n \cdot \frac{dn}{dx}$$
$$J_p^{(\mathrm{diff})} = +q \cdot \Phi_p = q \cdot \left(-D_p \cdot \frac{dp}{dx}\right) = -qD_p \cdot \frac{dp}{dx}$$

Einstein relations

The net currents are:

$$J_n = q\mu_n n\mathrm{E} + qD_n \cdot \frac{dn}{dx}, \; J_p = q\mu_p p\mathrm{E} - qD_p \cdot \frac{dp}{dx}$$

where the first term in each is the drift current and the second term is the diffusion current. The constants D_n and D_p are electron and hole diffusion constants, respectively.

The E-field represents a change in electrical potential w.r.t. position: $\mathrm{E} = -dV/dx$. Further note that the band edges indicate potential energies of electrons. For example, the energy of an electron at the conduction band edge is $E_\mathrm{C}(x) = E_{\mathrm{C}0} - qV(x)$. Using (), we see that the variation in $E_\mathrm{C}(x)$ indicates a corresponding variation in electron concentration

$$n(x) = N_\mathrm{C} \cdot \mathrm{e}^{-[E_\mathrm{C}(x) - E_\mathrm{F}]/kT}$$

The gradient in $n(x)$ is

$$\frac{dn}{dx} = -\left(\frac{1}{kT}\right) \cdot \frac{dE_\mathrm{C}}{dx} \cdot n(x) = \frac{-q\mathrm{E}}{kT} \cdot n(x)$$

In equilibrium, the electron current is zero, so:

$$J_n = 0 = \cancel{q}\mu_n \cancel{n} \cancel{\mathrm{E}} - \cancel{q} D_n \cdot \left(\frac{q\cancel{\mathrm{E}}}{kT}\right) \cdot \cancel{n}$$

The same principle applies for holes. This gives the Einstein relations:

$$\frac{D_n}{\mu_n} = \frac{kT}{q} \text{ and } \frac{D_p}{\mu_p} = \frac{kT}{q}$$

Continuity equation (I)

The continuity equation is a well-known principle from electrodynamics. Incorporating the generation rate, G_n, and recombination rate, U_n,

$$\frac{\partial n}{\partial t} = \frac{1}{q}\frac{\partial J_n}{\partial x} + G_n - U_n$$

Consider steady state: $\partial n/\partial t = 0$. So

$$0 = \frac{1}{q}\frac{dJ_n}{dx} + G_n - U_n$$

For holes:

$$\frac{\partial p}{\partial t} = -\frac{1}{q}\frac{\partial J_p}{\partial x} + G_p - U_p$$

In steady state: $\partial p/\partial t = 0$. So

$$0 = -\frac{1}{q}\frac{dJ_p}{dx} + G_p - U_p$$

Continuity equation (II)

In steady state, we can write for electrons

$$-\frac{1}{q}\frac{dJ_n}{dx} = G_n - U_n$$

This gives

$$\frac{dJ_n}{dx} = q\mu_n \cdot \frac{dn}{dx} \cdot \mathrm{E} + q\mu_n \cdot n \cdot \frac{d\mathrm{E}}{dx} + qD_n \cdot \frac{d^2 n}{dx^2}$$

leading to

$$-D_n \cdot \frac{d^2 n}{dx^2} - \mu_n \cdot \mathrm{E} \cdot \frac{dn}{dx} - \mu_n \cdot \frac{d\mathrm{E}}{dx} \cdot n = G_n - U_n$$

For holes in steady state

$$\frac{1}{q}\frac{dJ_p}{dx} = G_p - U_p$$

In this case

$$\frac{dJ_p}{dx} = q\mu_p \cdot \frac{dp}{dx} \cdot \mathrm{E} + q\mu_p \cdot p \cdot \frac{d\mathrm{E}}{dx} - qD_p \cdot \frac{d^2 p}{dx^2}$$

which gives

$$-D_p \cdot \frac{d^2 p}{dx^2} + \mu_p \cdot \mathrm{E} \cdot \frac{dp}{dx} + \mu_p \cdot \frac{d\mathrm{E}}{dx} \cdot p = G_p - U_p$$

We will return to these forms later.

Non-equilibrium, steady state

The equilibrium analysis presented to this point can encapsulate time-independent, non-equilibrium phenomena, the so-called steady state. This quasi-equilibrium situation is always present under illumination, or with applied, assuming sufficient time has elapsed for transient effects to diminish. In particular, we cannot assume equilibrium between the carrier populations in the CB and VB. However, we continue to assume equilibrium within these bands. Thus, we have Fermi levels in the two bands E_{F_n} and E_{F_p} , called quasi-Fermi levels, and temperatures T_n and T_p. However, equilibration of carriers in the bands with the lattice at temperature T eliminates any "hot" carriers, so we usually assume a uniform temperature $T_n \approx T_p = T$

$$f_n = \frac{1}{e^{(E-E_{F_n})/kT_n}+1} \rightarrow \frac{1}{e^{(E-E_{F_n})/kT}+1}$$
$$f_p = \frac{1}{e^{(E-E_{F_p})/kT_p}+1} \rightarrow \frac{1}{e^{(E-E_{F_p})/kT}+1}$$

separate states of equilibrium (distributions) within CB and VB. The carrier concentrations are

$$n = n_i \cdot e^{(E_{F_n}-E_i)/kT} = N_C \cdot e^{(E_C-E_{F_n})/kT} \text{ and } p = n_i \cdot e^{(E_i-E_{F_p})/kT} = N_V \cdot e^{(E_{F_p}-E_V)/kT}$$

The $n \cdot p$ product now depends on the quasi-Fermi-level splitting, according to the "law of mass action":

$$n \cdot p = n_i^2 \cdot e^{(E_{F_n}-E_{F_p})/kT} = n_i^2 \cdot e^{\Delta\mu/kT}$$

The chemical potential difference $\Delta\mu = E_{F_n} - E_{F_p}$ is a measure of the deviation from equilibrium.

Steady-state current

We know from equilibrium analysis that:

$$J_n = \mu_n \cdot \left(qn\mathrm{E} + kT \cdot \frac{dn}{dx}\right) \text{ and } J_p = \mu_p \cdot \left(qp\mathrm{E} - kT \cdot \frac{dp}{dx}\right)$$

In quasi-equilibrium, the carrier concentrations follow:

$$n(x) = n_i \cdot e^{[E_{F_n}(x)-E_i(x)]/kT} \text{ and } p(x) = n_i \cdot e^{[E_i(x)-E_{F_p}]/kT}$$

The presence of an electric field $\mathrm{E} = -dV/dx$ affects the intrinsic energy according to $E_i(x) = E_{i0} - qV(x)$. Using $dE_i/dx = q\mathrm{E}$:

$$kT \cdot \frac{dn}{dx} = \left(\frac{dE_{F_n}}{dx} - \frac{dE_i}{dx}\right) \cdot n = \left(\frac{dE_{F_n}}{dx} - qE\right) \cdot n$$
$$kT \cdot \frac{dp}{dx} = \left(\frac{dE_i}{dx} - \frac{dE_{F_p}}{dx}\right) \cdot p = \left(qE - \frac{dE_{F_p}}{dx}\right) \cdot p$$

These provide simple forms for current densities in terms of gradients in E_{F_n} and E_{F_p} :

$$J_n = \mu_n \cdot n \cdot \frac{dE_{F_n}}{dx} \text{ and } J_p = \mu_p \cdot p \cdot \frac{dE_{F_p}}{dx}$$

Diffusion length

Assume we have non-equilibrium carrier concentrations $n = n_0 + \Delta n$ and $p = p_0 + \Delta p$, where n_0 and p_0 are the equilibrium concentrations and Δn and Δp are the excess carrier concentrations. The recombination rate is often proportional to the excess carrier concentration:

$$U_n = \frac{dn}{dt} = \frac{d(\Delta n)}{dt} = \frac{\Delta n}{\tau_n} = \frac{n - n_0}{\tau_n} \text{ and } U_p = \frac{dp}{dt} = \frac{d(\Delta p)}{dt} = \frac{\Delta p}{\tau_p} = \frac{p - p_0}{\tau_p}$$

In steady-state, for electrons

$$\frac{\partial n}{\partial t} = 0 = \frac{1}{q}\frac{dJ_n}{dx} + G_n - U_n$$

Using our previous result

$$0 = D_n \cdot \frac{d^2 n}{dx^2} + \mu_n \cdot E \cdot \frac{dn}{dx} + \mu_n \cdot \frac{dE}{dx} \cdot n + G_n - U_n$$

Assume we are in a region with no electric field ($\mathrm{E} = 0$ and $d\mathrm{E}/dx = 0$) and no generation $G_n = 0$. Then

$$\frac{d^2 n}{dx^2} = \frac{n - n_0}{D_n \cdot \tau_n} = \frac{n - n_0}{L_n^{\ 2}}$$

We have defined the *diffusion length* for electrons $L_n = \sqrt{D_n \cdot \tau_n}$.

Diffusion problem

Let's consider a problem in which diffusion length is important. The general solution for the preceding second-order differential equation is

$$\Delta n(x) = A \cdot \mathrm{e}^{x/L_n} + B \cdot \mathrm{e}^{-x/L_n} = (A + B) \cdot \cosh\left(\frac{x}{L_n}\right) + (A - B) \cdot \sinh\left(\frac{x}{L_n}\right)$$

In this example, let's assume the excess carrier concentration is fixed to be Δn_b at boundaries $x = \pm x_0$. The solution must satisfy

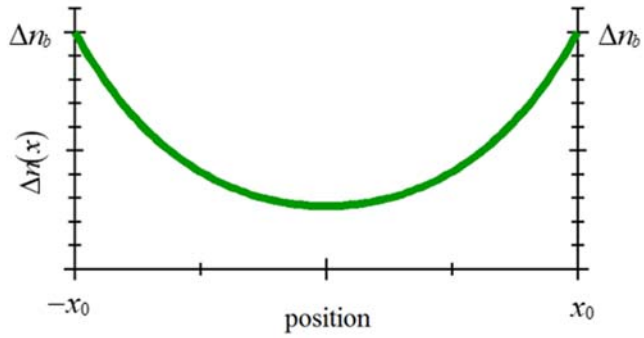
$$\begin{aligned} \Delta n_b &= (A + B) \cdot \cosh\left(\frac{x_0}{L_n}\right) - (A - B) \cdot \sinh\left(\frac{x_0}{L_n}\right) \\ &= (A + B) \cdot \cosh\left(\frac{x_0}{L_n}\right) + (A - B) \cdot \sinh\left(\frac{x_0}{L_n}\right) \end{aligned}$$

Clearly $A = B$, so

$$A + B = \frac{\Delta n_b}{\cosh\left(\frac{x_0}{L_n}\right)}$$

Now

$$\Delta n(x) = \Delta n_b \cdot \frac{\cosh\left(\frac{x}{L_n}\right)}{\cosh\left(\frac{x_0}{L_n}\right)}$$



We might be interested in the excess carrier concentration at $x = 0$:

$$\Delta n(0) = \frac{\Delta n_b}{\cosh\left(\frac{x_0}{L_n}\right)}$$