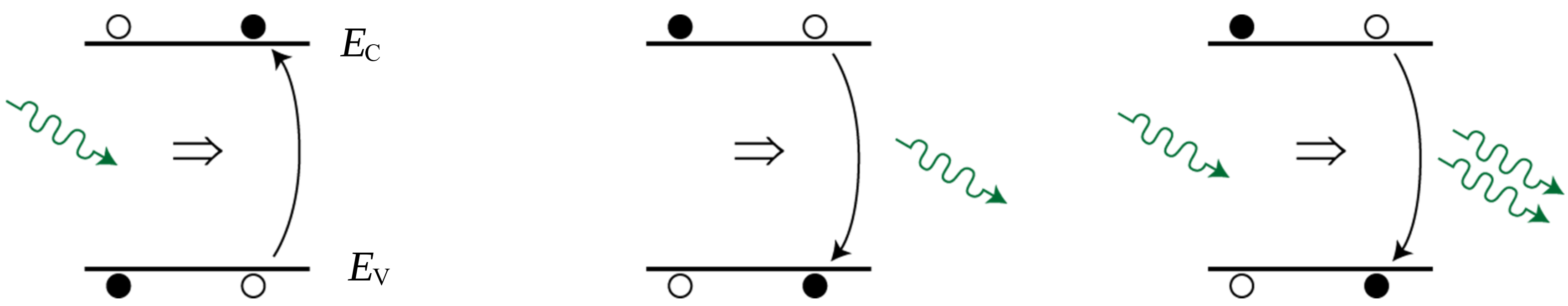
Band-to-band optical processes (I)

Two state system:

1) absorption *2) spontaneous emission* *3) stimulated emission*



$E_{ph} = E_C - E_V = E_g$ //photon energy

Distribution functions:

$$f_C = f\left(E_C - E_{F_n}\right) = \frac{1}{e^{(E_C - E_{F_n})/kT} + 1}$$

$$1 - f_V = f\left(E_{F_p} - E_V\right) = \frac{1}{e^{(E_{F_p} - E_V)/kT} + 1}$$

Probabilities

$P_{hole} = 1 - f_V$ $P_{electron} = f_C$

$\bar{P}_{hole} = f_V$ $\bar{P}_{electron} = 1 - f_C$

Rates

1) $r_1 = A \cdot f_{ph} \cdot f_V \cdot (1 - f_C)$

2) $r_2 = B \cdot (1 - f_V) \cdot f_C$

3) $r_3 = C \cdot f_{ph} \cdot (1 - f_V) \cdot f_C$

Band-to-band optical processes (II)

absorption ↘ $r_1 = \overbrace{r_2 + r_3}^{\textit{emission}}$ //equilibrium

$$A \cdot f_{\mathrm{ph}} \cdot f_{\mathrm{V}} \cdot (1 - f_{\mathrm{C}}) = B \cdot (1 - f_{\mathrm{V}}) \cdot f_{\mathrm{C}} + C \cdot f_{\mathrm{ph}} \cdot (1 - f_{\mathrm{V}}) \cdot f_{\mathrm{C}}$$

Q: What is f_{ph} needed to maintain equilibrium?

$$f_{\mathrm{ph}} = \frac{B \cdot (1 - f_{\mathrm{V}}) \cdot f_{\mathrm{C}}}{A \cdot f_{\mathrm{V}} \cdot (1 - f_{\mathrm{C}}) - C \cdot (1 - f_{\mathrm{V}}) \cdot f_{\mathrm{C}}} = \frac{B}{A} \cdot \left[\frac{1}{\left(\frac{\frac{1}{f_{\mathrm{C}}} - 1}{\frac{1}{f_{\mathrm{V}}} - 1} \right) - \frac{C}{A}} \right]$$

$$\frac{1}{f_{\mathrm{C}}} - 1 = \mathrm{e}^{(E_{\mathrm{C}} - E_{\mathrm{F}_n})/kT}$$

$$\frac{1}{f_{\mathrm{V}}} - 1 = \mathrm{e}^{(E_{\mathrm{F}_p} - E_{\mathrm{V}})/kT}$$

$$\frac{\frac{1}{f_{\mathrm{C}}} - 1}{\frac{1}{f_{\mathrm{V}}} - 1} = \mathrm{e}^{(E_g - \Delta\mu)/kT}$$

$$\Delta\mu = E_{\mathrm{F}_n} - E_{\mathrm{F}_p}$$

$$f_{\mathrm{ph}} = \frac{B/A}{\mathrm{e}^{(E_g - \Delta\mu)/kT} - C/A}$$

Band-to-band optical processes (III)

Recall, for blackbody radiation (empty cavity filled w/radiation):

$$f_{\mathrm{ph}} = \frac{1}{\mathrm{e}^{E/kT} - 1}$$

//Bose-Einstein distribution function

Our formulation must reduce to this form when the solid is absent:

$$\Rightarrow A = B = C$$

In the presence of a two-state system (e.g., a semiconductor):

$$\Rightarrow E \rightarrow E_g - \Delta\mu$$

$$\Rightarrow f_{\mathrm{ph}} = \frac{1}{\mathrm{e}^{(E_g - \Delta\mu)/kT} - 1}$$

The radiation field is altered by the semiconductor.
There is a “photon chemical potential”, equal to the quasi-Fermi-level splitting.

Radiative Recombination

For spontaneous emission:

$$U_{\mathrm{sp}} \propto B_{\mathrm{rad}} \cdot n \cdot p$$

$$U_{\mathrm{rad}} = U_{\mathrm{sp}} - U_{\mathrm{sp}}^{(0)} = B_{\mathrm{rad}} \cdot \left(n \cdot p - n_0 \cdot p_0\right) \approx B_{\mathrm{rad}} \cdot {n_i}^2 \cdot \left(\mathrm{e}^{\Delta\mu/kT} - 1\right)$$

$$n = n_0 + \Delta n$$

$$p = p_0 + \Delta p$$

$$n \cdot p - n_0 \cdot p_0 \approx n_0 \cdot \Delta p + p_0 \cdot \Delta n$$

In doped material, we have:

$$n_0 \gg p_0, \qquad \text{(n-type)}$$

$$p_0 \gg n_0, \qquad \text{(p-type)}$$

So:

excess minority carrier conc. ↓

$$U_{\mathrm{rad}} = B_{\mathrm{rad}} \cdot \begin{cases} n_0 \cdot \Delta p, & \text{//n-type} \\ p_0 \cdot \Delta n, & \text{//p-type} \end{cases}$$

We can write:

$$U_{\mathrm{rad}} = B_{\mathrm{rad}} \cdot \begin{cases} \dfrac{\Delta p}{\tau_{p,\mathrm{rad}}}, & \text{//n-type} \\ \dfrac{\Delta n}{\tau_{n,\mathrm{rad}}}, & \text{//p-type} \end{cases}$$

Assume:

$$n_0 = N_{\mathrm{D}}, \qquad \text{//n-type}$$

$$p_0 = N_{\mathrm{A}}, \qquad \text{//p-type}$$

$$\tau_{p,\mathrm{rad}} = \frac{1}{B_{\mathrm{rad}} \cdot N_{\mathrm{D}}}, \qquad \text{//n-type}$$

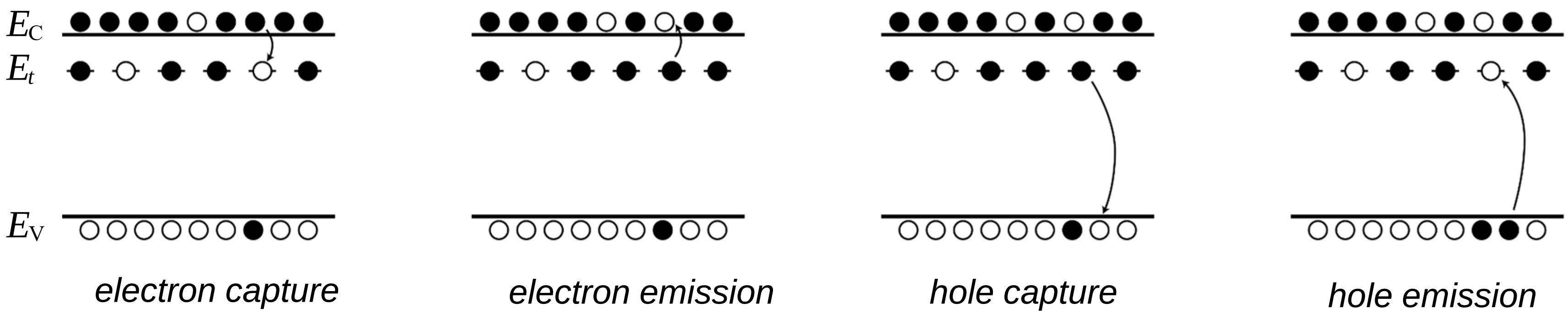
$$\tau_{n,\mathrm{rad}} = \frac{1}{B_{\mathrm{rad}} \cdot N_{\mathrm{A}}}, \qquad \text{//p-type}$$

//minority-carrier radiative lifetimes

If $\Delta\mu$ results from applied bias: $U_{\mathrm{rad}} = B_{\mathrm{rad}} \cdot {n_i}^2 \cdot \left(\mathrm{e}^{qV/kT} - 1\right)$

Shockley-Read-Hall recombination (I)

Trap states within the bandgap:



trap density

trap occupation probability

electron capture rate: $U_n = B_n \cdot N_t \cdot n \cdot (1 - f_t) = \frac{n \cdot (1 - f_t)}{\tau_n}$

electron emission rate: $G_n = \frac{N_t \cdot f_t}{\tau_{n,\mathrm{esc}}}$ $\tau_n = \frac{1}{B_n \cdot N_t}$

hole capture rate: $U_p = B_p \cdot N_t \cdot p \cdot f_t = \frac{p \cdot f_t}{\tau_p}$

hole emission rate: $G_p = \frac{N_t \cdot (1 - f_t)}{\tau_{p,\mathrm{esc}}}$ $\tau_p = \frac{1}{B_p \cdot N_t}$

$B_n = v_n \cdot \sigma_n$

$B_p = v_p \cdot \sigma_p$

thermal velocity

capture cross section

Shockley-Read-Hall recombination (II)

In equilibrium: $G_n = U_n$ $\quad B_n \cdot \cancel{N_t} \cdot n_0 \cdot (1 - f_t) = \frac{\cancel{N_t} \cdot f_t}{\tau_{n,esc}}$ $\quad \frac{1}{\tau_{n,\mathrm{esc}}} = B_n \cdot n_0 \cdot \left(\frac{1}{f_t} - 1 \right)$

$$f_t = \frac{1}{\mathrm{e}^{(E_t - E_\mathrm{F})/kT} + 1} \qquad \frac{1}{f_t} - 1 = \mathrm{e}^{(E_t - E_\mathrm{F})/kT} \qquad \frac{1}{\tau_{n,\mathrm{esc}}} = B_n \cdot n_0 \cdot \mathrm{e}^{(E_t - E_\mathrm{F})/kT}$$

$$n_0 = n_i \cdot \mathrm{e}^{(E_\mathrm{F} - E_i)/kT}$$

$n_t \doteq n_0 \cdot \mathrm{e}^{(E_t - E_\mathrm{F})/kT} = n_i \cdot \mathrm{e}^{(E_t - E_i)/kT}$ //free electron density when: $E_\mathrm{F} = E_t$

$\frac{1}{\tau_{n,esc}} = B_n \cdot n_t$ $\quad \Rightarrow G_n = N_t \cdot f_t \cdot B_n \cdot n_t$ $\quad$ Use: $\frac{1}{\tau_n} = B_n \cdot N_t$ $\quad \Rightarrow G_n = \frac{n_t \cdot f_t}{\tau_n}$

Also: $G_p = U_p$ $\quad B_p \cdot \cancel{N_t} \cdot p_0 \cdot f_t = \frac{\cancel{N_t} \cdot (1 - f_t)}{\tau_{p,\mathrm{esc}}}$ $\quad \frac{1}{\tau_{p,\mathrm{esc}}} = B_p \cdot p_0 \cdot \left(\frac{1}{\frac{1}{f_t} - 1} \right)$

$$\frac{1}{\frac{1}{f_t} - 1} = \mathrm{e}^{(E_\mathrm{F} - E_t)/kT} \qquad \frac{1}{\tau_{p,\mathrm{esc}}} = B_p \cdot p_0 \cdot \mathrm{e}^{(E_\mathrm{F} - E_t)/kT}$$

$$p_0 = p_i \cdot \mathrm{e}^{(E_i - E_\mathrm{F})/kT}$$

$p_t \doteq p_0 \cdot \mathrm{e}^{(E_\mathrm{F} - E_t)/kT} = n_i \cdot \mathrm{e}^{(E_i - E_t)/kT}$ //free hole density when: $E_\mathrm{F} = E_t$

$\frac{1}{\tau_{p,\mathrm{esc}}} = B_p \cdot p_t$ $\quad \Rightarrow G_p = N_t \cdot (1 - f_t) \cdot B_p \cdot p_t$ $\quad$ Use: $\frac{1}{\tau_p} = B_p \cdot N_t$ $\quad \Rightarrow G_p = \frac{p_t \cdot (1 - f_t)}{\tau_p}$

Shockley-Read-Hall recombination (III)

In steady-state: $U_{\text{SRH}} = U_n - G_n = U_p - G_p$

$$\frac{n \cdot (1 - f_t)}{\tau_n} - \frac{N_t \cdot f_t}{\tau_{n,esc}} = \frac{p \cdot f_t}{\tau_p} - \frac{N_t \cdot (1 - f_t)}{\tau_{p,esc}}$$

$$B_n = \frac{1}{N_t \cdot \tau_n} \quad \Rightarrow \frac{1}{\tau_{n,\text{esc}}} = B_n \cdot n_t = \frac{n_t}{N_t \cdot \tau_n} \qquad B_p = \frac{1}{N_t \cdot \tau_p} \quad \Rightarrow \frac{1}{\tau_{p,esc}} = B_p \cdot p_t = \frac{p_t}{N_t \cdot \tau_p}$$

From equilibrium analysis:

$$\frac{n \cdot (1 - f_t)}{\tau_n} - \frac{n_t \cdot f_t}{\tau_n} = \frac{p \cdot f_t}{\tau_p} - \frac{p_t \cdot (1 - f_t)}{\tau_p} \qquad \left(\frac{p + p_t}{\tau_p} + \frac{n + n_t}{\tau_n} \right) \cdot f_t = \frac{n}{\tau_n} + \frac{p_t}{\tau_p}$$

$$f_t = \frac{\dfrac{n}{\tau_n} + \dfrac{p_t}{\tau_p}}{\dfrac{p + p_t}{\tau_p} + \dfrac{n + n_t}{\tau_n}} = \frac{n \cdot \tau_p + p_t \cdot \tau_n}{(n + n_t) \cdot \tau_p + (p + p_t) \cdot \tau_n}$$

//probability that trap is filled

$$1 - f_t = \frac{n_t \cdot \tau_p + p \cdot \tau_n}{(n + n_t) \cdot \tau_p + (p + p_t) \cdot \tau_n}$$

//probability that trap is empty

Shockley-Read-Hall recombination (IV)

$$U_{\mathrm{SRH}} = U_p - G_p \left(= U_n - G_n\right) = \frac{p \cdot f_t}{\tau_p} - \frac{p_t \cdot \left(1 - f_t\right)}{\tau_p} = \frac{n \cdot p - n_t \cdot p_t}{\left(n + n_t\right) \cdot \tau_p + \left(p + p_t\right) \cdot \tau_n}$$

Recall: $n \cdot p = n_i^2 \mathrm{e}^{\Delta\mu/kT}$ $\qquad n_t \cdot p_t = n_i^2$

$$U_{\mathrm{SRH}} = \frac{n_i^2 \cdot \left(\mathrm{e}^{\Delta\mu/kT} - 1\right)}{\left(n + n_t\right) \cdot \tau_p + \left(p + p_t\right) \cdot \tau_n}$$

In terms of excess carrier concentrations:

$$n = n_0 + \Delta n$$

$$p = p_0 + \Delta p$$

$$\begin{aligned} n \cdot p - n_t \cdot p_t &= n \cdot p - n_i^2 \\ &= n \cdot p - n_0 \cdot p_0 \\ &\approx n_0 \cdot \Delta p + p_0 \cdot \Delta n \end{aligned}$$

$$U_{\mathrm{SRH}} = \frac{n_0 \cdot \Delta p + p_0 \cdot \Delta n}{\left(n_0 + \Delta n + n_t\right) \cdot \tau_p + \left(p_0 + \Delta p + p_t\right) \cdot \tau_n}$$

Consider moderately doped material:

$$n_0 \gg p_0, \qquad \text{//n-type}$$

$$p_0 \gg n_0, \qquad \text{//p-type}$$

$$U_{\mathrm{SRH}} = \begin{cases} \dfrac{\Delta p}{\tau_p}, & \text{//n-type} \\ \dfrac{\Delta n}{\tau_n}, & \text{//p-type} \end{cases}$$

Shockley-Read-Hall recombination (V)

Consider n-type material, moderate doping: $n_0 = N_\mathrm{D}$

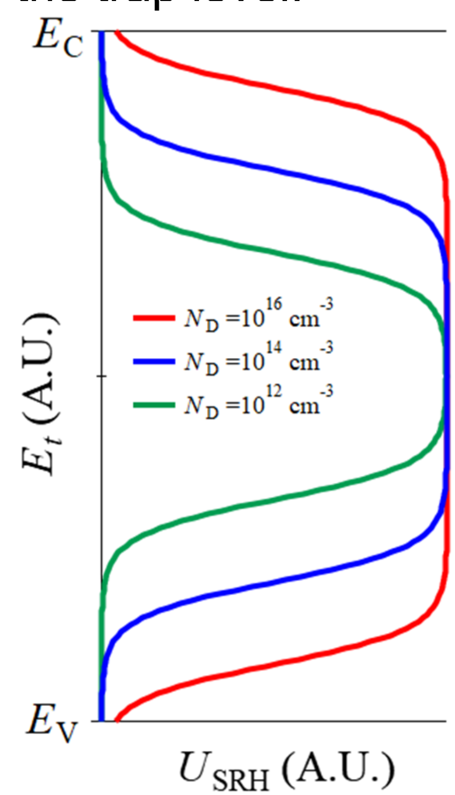
Compare the recombination rate for various locations of the trap level:

$$U_\mathrm{SRH} = \frac{N_\mathrm{D}\cdot\Delta p + \cancel{\left(\frac{n_i^2}{N_\mathrm{D}}\right)\cdot\Delta n}}{\left(N_\mathrm{D} + \cancel{\Delta n} + n_t\right)\cdot\tau_p + \left(\cancel{\frac{n_i^2}{N_\mathrm{D}}} + \Delta p + \frac{n_i^2}{n_t}\right)\cdot\tau_n}$$

$$\approx \frac{N_\mathrm{D}\cdot n_t\cdot\Delta p}{n_t^2\cdot\tau_p + \left(N_\mathrm{D}\cdot\tau_p + \cancel{\Delta p\cdot\tau_n}\right)\cdot n_t + n_i^2\cdot\tau_n}$$

$$U_\mathrm{SRH} \approx \frac{\Delta p}{\tau_p + \left(n_t\cdot\tau_p + p_t\cdot\tau_n\right)/N_\mathrm{D}}$$

Assume lifetimes are of the same magnitude.



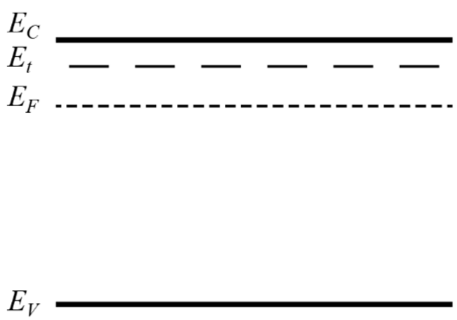
$$N_\mathrm{D} = n_i\cdot e^{(E_\mathrm{F}-E_i)/kT} \qquad n_t = n_i\cdot e^{(E_t-E_i)/kT} \qquad p_t = n_i\cdot e^{-(E_t-E_i)/kT}$$

Shockley-Read-Hall recombination (VI)

Assume n-type:

1) Shallow electron trap (near CB edge):

$$E_t > E_F$$
$$n_t \gg N_D$$

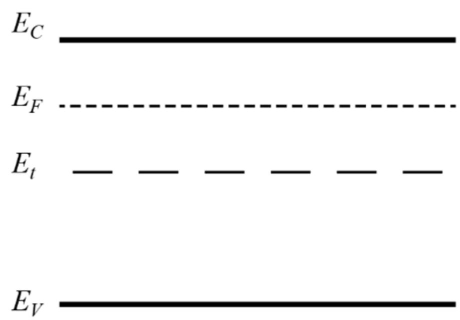


$$U_{SRH} = \frac{N_D \cdot \Delta p}{n_t \cdot \tau_p}$$

$$\frac{1}{\tau} = \frac{N_D}{n_t \cdot \tau_p} \left(\ll \frac{1}{\tau_p} \right)$$

2) Deep level (mid-gap):

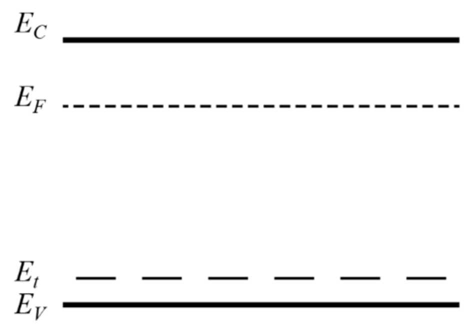
$$E_t < E_F$$
$$N_D \gg n_t$$



$$U_{SRH} = \frac{\Delta p}{\tau_p}$$

3) Shallow hole trap (near VB edge):

$$E_t < E_F$$
$$p_t \gg N_D$$



$$U_{SRH} = \frac{N_D \cdot \Delta p}{p_t \cdot \tau_n}$$

$$\frac{1}{\tau} = \frac{N_D}{p_t \cdot \tau_n} \left(\ll \frac{1}{\tau_p} \right)$$

SRH recombination rate is highest when the trap level is midgap.

Auger Recombination

$$U_{p,Aug} = A_p \cdot \left(n^2 \cdot p - n_0^{\ 2} \cdot p_0\right)$$

$$U_{n,Aug} = A_n \cdot \left(n \cdot p^2 - n_0 \cdot p_0^{\ 2}\right)$$

p-type: $p \approx p_0 \approx N_a$

$$U_{n,Aug} = A_n \cdot \left[(n_0 + \Delta n) \cdot N_a^{\ 2} - n_0 \cdot N_a^{\ 2}\right]$$

$$\approx A_n \cdot N_a^{\ 2} \cdot \Delta n = \frac{\Delta n}{\tau_{n,Aug}}$$

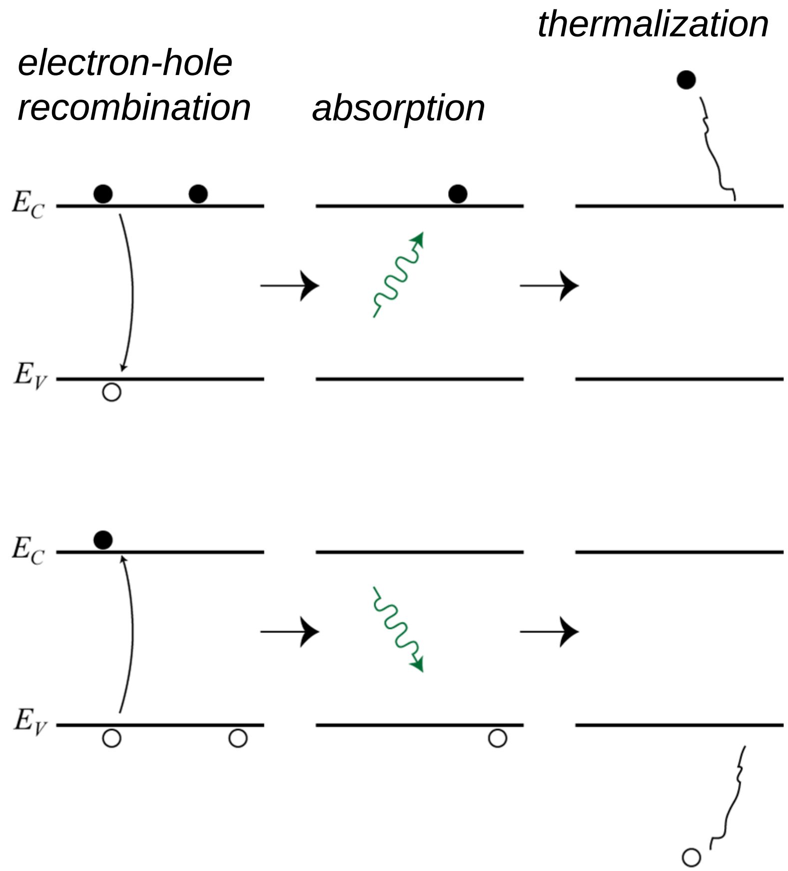
$$\frac{1}{\tau_{n,Aug}} = A_n \cdot N_a^{\ 2}$$

n-type: $n \approx n_0 \approx N_d$

$$U_{p,Aug} = A_p \cdot \left[N_d^{\ 2} \cdot (p_0 + \Delta p) - N_d^{\ 2} \cdot p_0\right]$$

$$\approx A_p \cdot N_p^{\ 2} \cdot \Delta p = \frac{\Delta p}{\tau_{p,Aug}}$$

$$\frac{1}{\tau_{p,Aug}} = A_p \cdot N_d^{\ 2}$$



Net Lifetime

n-type:

$$\Delta p(t) = \Delta p(0) \cdot e^{-t/\tau_{p,SRH}} \cdot e^{-t/\tau_{p,rad}} \cdot e^{-t/\tau_{p,Aug}} = \Delta p(0) \cdot e^{-t/\tau_p}$$

$$\frac{1}{\tau_p} = \frac{1}{\tau_{p,SRH}} + \frac{1}{\tau_{p,rad}} + \frac{1}{\tau_{p,Aug}} = B_p \cdot N_t + B_{rad} \cdot N_d + A_{p,Aug} \cdot N_d^{\,2}$$

p-type:

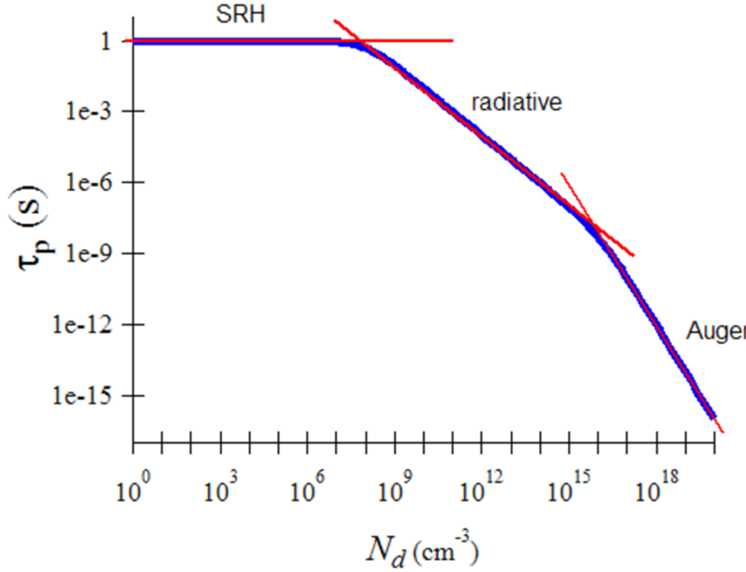
$$\Delta n(t) = \Delta n(0) \cdot e^{-t/\tau_{SRH}} \cdot e^{-t/\tau_{rad}} \cdot e^{-t/\tau_{Aug}} = \Delta n(0) \cdot e^{-t/\tau}$$

$$\frac{1}{\tau_n} = \frac{1}{\tau_{n,SRH}} + \frac{1}{\tau_{n,rad}} + \frac{1}{\tau_{n,Aug}} = B_n \cdot N_t + B_{rad} \cdot N_a + A_{n,Aug} \cdot N_a^{\,2}$$

no doping: SRH dominates

moderate doping: radiative dominates

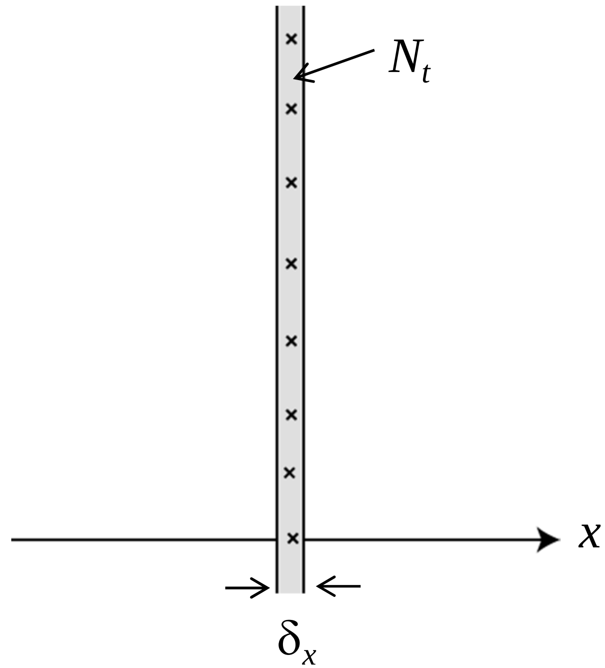
high doping: Auger dominates



Surface Recombination (I)

Consider SRH centers localized at a 2-D interface

$$U_{\mathrm{SRH}} \cdot \delta_x = \frac{n \cdot p - n_i^2}{\frac{\tau_n}{\delta_x} \cdot (p + p_t) + \frac{\tau_p}{\delta_x} \cdot (n + n_t)}$$



The areal density of traps is: $N_s = N_t \cdot \delta_x$

$[N_t] = \mathrm{cm}^{-3}$ $[N_s] = \mathrm{cm}^{-2}$

$$\frac{1}{\tau_n} = B_n \cdot N_t = B_n \cdot \frac{N_s}{\delta_x} \qquad \frac{1}{\tau_p} = B_p \cdot N_t = B_p \cdot \frac{N_s}{\delta_x}$$

$$\frac{\delta_x}{\tau_n} = B_n \cdot N_s = S_n \qquad \frac{\delta_x}{\tau_p} = B_p \cdot N_s = S_p$$

//Surface recombination “velocities”

$$U_{SRH} \cdot \delta_x = \frac{n \cdot p - n_i^2}{\frac{1}{S_n} \cdot (p + p_t) + \frac{1}{S_p} \cdot (n + n_t)}$$

$$[S_n] = [S_p] = \frac{\mathrm{cm}}{\mathrm{s}}$$

Surface Recombination (II)

p-type: $p_0 = N_a(\gg n_0,\, n_i)$

$$n = n_0 + \Delta n \qquad p \approx p_0 = N_\mathrm{A}$$

$$n \cdot p - n_i^2 \approx (n_0 + \Delta n) \cdot p_0 - n_i^2 = \Delta n \cdot N_\mathrm{A}$$

$$\frac{1}{S_n} \cdot (p + p_t) + \frac{1}{S_p} \cdot (n + n_t) \approx \frac{N_\mathrm{A}}{S_n}$$

$$U_\mathrm{SRH} \cdot \delta_x \to S_n \cdot \Delta n$$

continuity, steady-state: $\frac{\partial n}{\partial t} = 0 = \frac{1}{q} \cdot \frac{dJ_n}{dx} + G_n - U_n$

$$\frac{dJ_n}{dx} = q \cdot (U_n - G_n) \qquad \frac{\delta J_n}{\delta x} = q \cdot \left(\frac{S_n \cdot \Delta n}{\delta_x} - G_n \right)$$

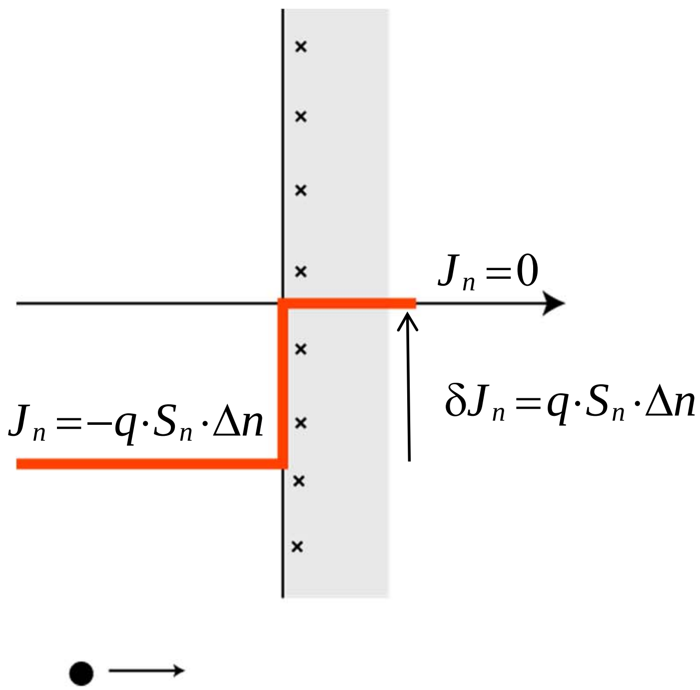
no generation not localized: $G_n \cdot \delta x \to 0$

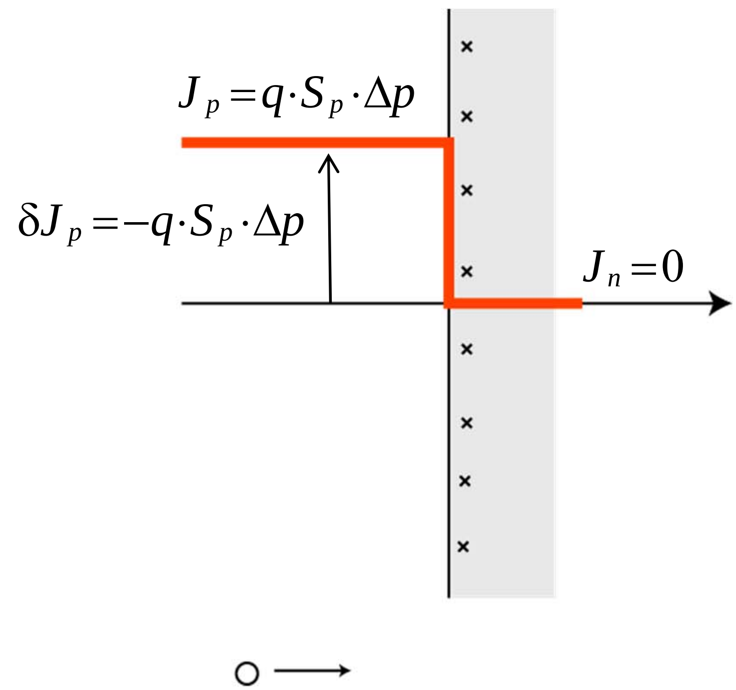
$$\delta J_n = q \cdot S_n \cdot \Delta n = J_n\left(x + \frac{\delta_x}{2}\right) - J_n\left(x - \frac{\delta_x}{2}\right)$$

$$J_n\big|_+ = J_n\big|_- + q \cdot S_n \cdot \Delta n$$

discontinuity in current

Surface Recombination (III)





At a surface:

$$J_n = -\delta J_n = -q \cdot S_n \cdot \Delta n$$

$$J_p = -\delta J_p = q \cdot S_p \cdot \Delta p$$

When the current drops to zero across an interface, the difference is the surface recombination current, which indicates the total current.

Surface Recombination (IV)

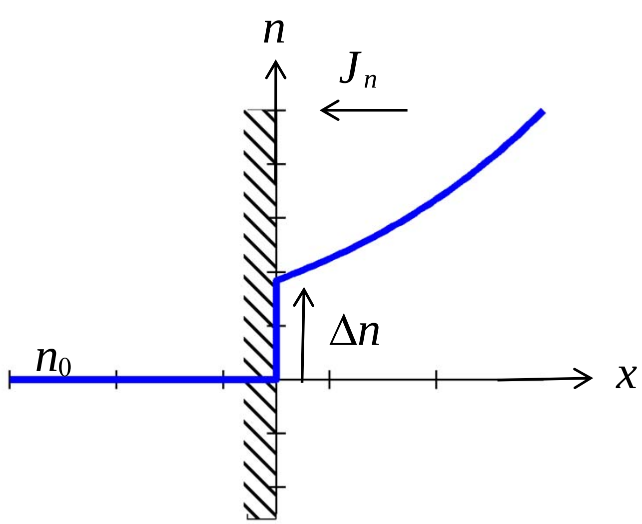
If no drift ($E=0$), current is diffusion only.

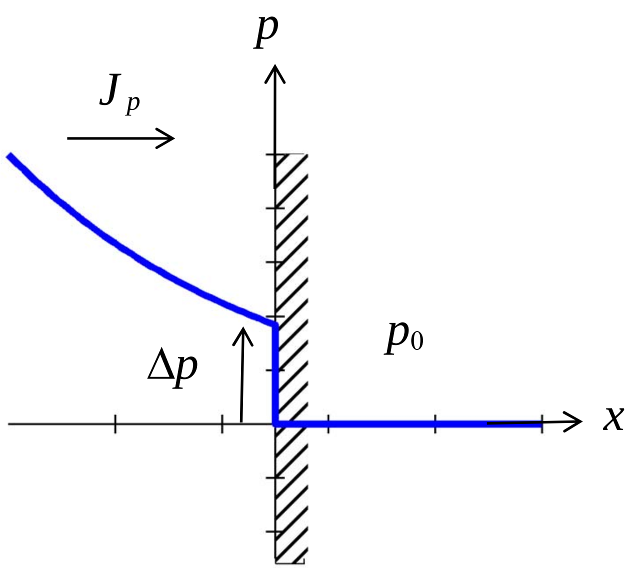
$$J_n\big|_{\text{interface}}=q\cdot D_n\cdot\frac{dn}{dx}=q\cdot D_n\cdot\frac{d(\Delta n)}{dx}=-q\cdot S_n\cdot\Delta n$$

$$\frac{d(\Delta n)}{dx}=-\frac{S_n}{D_n}\cdot\Delta n$$

$$J_p\big|_{\text{interface}}=-q\cdot D_p\cdot\frac{dp}{dx}=-q\cdot D_p\cdot\frac{d(\Delta p)}{dx}=q\cdot S_p\cdot\Delta p$$

$$\frac{d(\Delta p)}{dx}=-\frac{S_p}{D_p}\cdot\Delta p$$





Diffusion length (with generation)

$$\frac{\partial p}{\partial t} = -\frac{1}{q} \cdot \frac{\partial J_p}{\partial x} + G_p - U_p \,(=0)$$

//Continuity Eqn: (steady state)

$$\frac{dJ_p}{dx} = q \cdot \left(G_p - U_p\right)$$

recombination: $U_p(x) = \frac{p(x) - p_0}{\tau_p}$

neutral:

$$J_p = -q \cdot D_p \cdot \frac{dp}{dx}$$

define: $p_g = G_p \cdot \tau_p$

$$D_p \cdot \frac{d^2 p}{dx^2} = U_p - G_p$$

$$p(x) = p_0 + \Delta p(x) + p_g$$
$$= p_0 + \delta p(x)$$

$$p_0 = \frac{n_i^{\,2}}{N_d}$$

$$\frac{d^2 p}{dx^2} = \frac{p(x) - p_0 - p_g}{L_p^{\,2}}$$

diffusion length: $L_p = \sqrt{D_p \cdot \tau_p}$

$$\frac{d^2(\Delta p)}{dx^2} = \frac{\Delta p}{L_p^{\,2}}$$

$$\Delta p(x) = A \cdot \mathrm{e}^{x/L_p} + B \cdot \mathrm{e}^{-x/L_p}$$

"Effective" surface-recombination velocity

Say $\lim(x \to \infty)\left[\Delta p(x)\right] = 0$ $A = 0$ $\Delta p(x) = \Delta p(0) \cdot \mathrm{e}^{-x/L_p}$

The diffusion current at any point is:

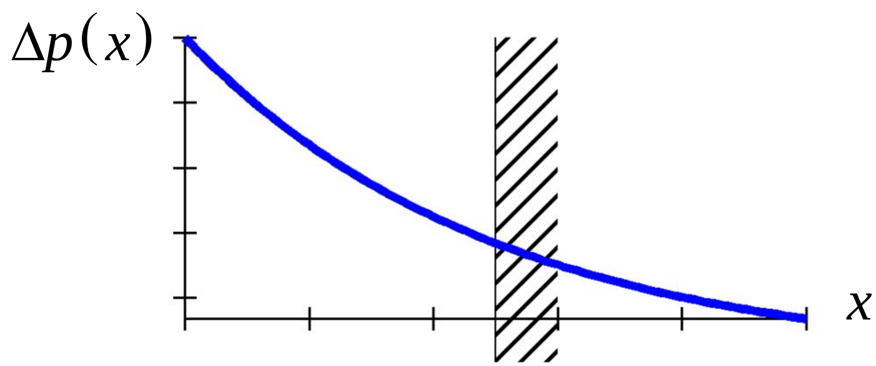
$$J_p(x) = -q \cdot D_p \cdot \frac{d(\Delta p)}{dx} = \frac{q \cdot D_p}{L_n} \cdot \Delta p(0) \cdot \mathrm{e}^{-x/L_p} = \frac{q \cdot D_p}{L_n} \cdot \Delta p(x)$$

This has the form of a surface recombination current:

$$J_p(x) = q \cdot S_{eff} \cdot \Delta p(x)$$

with an "effective"
surface recombination velocity:

$$S_{eff} = \frac{D_p}{L_p}$$



Surface Recombination: Example I

$$p(x) = p_0 + p_g + \Delta p(x)$$

Assume we know $\Delta p(0)$ and $\Delta p(w)$

$$\Delta p(x) = A \cdot \mathrm{e}^{x/L_p} + B \cdot \mathrm{e}^{-x/L_p}$$

$$\Delta p(0) = A + B \qquad \Delta p(w) = A \cdot \mathrm{e}^{w/L_p} + B \cdot \mathrm{e}^{-w/L_p}$$

$$\Delta p(x) = \frac{\Delta p(0) \cdot \sinh\left[(w-x)/L_p\right] + \Delta p(w) \cdot \sinh\left(x/L_p\right)}{\sinh\left(x/L_p\right)}$$

$$J_p(x) = -q \cdot D_p \cdot \frac{d(\Delta p)}{dx} = \frac{q \cdot D_p}{L_p}\left[\frac{\Delta p(0) \cdot \cosh\left[(w-x)/L_p\right] - \Delta p(w) \cdot \cosh\left(x/L_p\right)}{\sinh\left(w/L_p\right)}\right]$$

Surface Recombination: Example II

Assume we know $\Delta p(0)$ and $J_p(w) = q \cdot S_p \cdot \Delta p(w)$

$$\Delta p(0) = p(0) - p_0 \qquad \Delta p(w) = p(w) - p_0$$

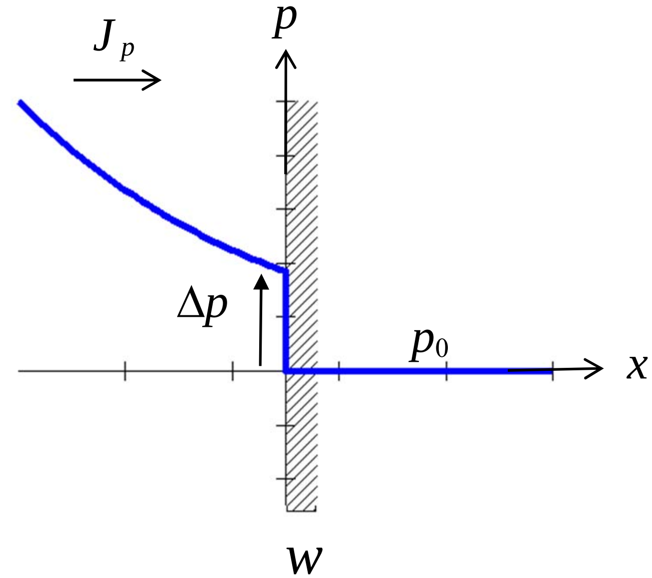
$$\frac{\not{q} \cdot D_p}{L_p} \cdot \left[\frac{\Delta p(0) - \Delta p(w) \cdot \cosh\left(w/L_p\right)}{\sinh\left(w/L_p\right)} \right] = \not{q} \cdot S_p \cdot \Delta p(w)$$

$$\Delta p(w) = \frac{\Delta p(0)}{\cosh\left(w/L_p\right) + r \cdot \sinh\left(w/L_p\right)}$$

$$\Delta p(x) = \frac{\Delta p(0) \cdot \left\{ \cosh\left[(w-x)/L_p\right] + r \cdot \sinh\left[(w-x)/L_p\right] \right\}}{\cosh\left(w/L_p\right) + r \cdot \sinh\left(w/L_p\right)}$$

$$J_p(x) = -q \cdot D_p \cdot \left.\frac{d(\Delta p)}{dx}\right|_x$$

$$= \frac{q \cdot D_p}{L_p} \cdot \frac{\left\{ \sinh\left[(w-x)/L_p\right] + r \cdot \cosh\left[(w-x)/L_p\right] \right\}}{\cosh\left(w/L_p\right) + r \cdot \sinh\left(w/L_p\right)} \cdot \Delta p(0)$$



$$r = \frac{S_p \cdot L_p}{D_p}$$

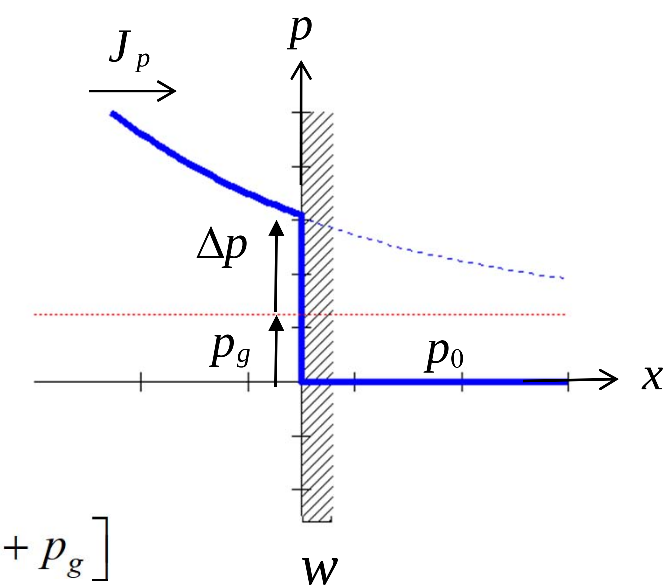
Surface Recombination: Example III

p/n junction solar cell, with recombination at back surface:

We know $\Delta p(0) = p_0 \cdot \left(e^{qV/kT} - 1\right) - p_g = -p_g \cdot (1 - R)$ $\quad R = p_0 \cdot \left(e^{qV/kT} - 1\right) / p_g$

and $J_p(w) = q \cdot S_p \cdot \left[p(w) - p_0 \right]$

$\Delta p(0) = p(0) - p_0 - p_g \qquad \Delta p(w) = p(w) - p_0 - p_g$



We require:

$$J_p(w) = q \cdot S_p \cdot \left[\Delta p(w) + p_g \right]$$

$$\frac{\not{q} \cdot D_p}{L_p} \cdot \left[\frac{\Delta p(0) - \Delta p(w) \cdot \cosh\left(w/L_p\right)}{\sinh\left(w/L_p\right)} \right] = \not{q} \cdot S_p \cdot \left[\Delta p(w) + p_g \right]$$

$$\Delta p(w) = \frac{\Delta p(0) - r \cdot \sinh\left(w/L_p\right) \cdot p_g}{\cosh\left(w/L_p\right) + r \cdot \sinh\left(w/L_p\right)} \qquad r = \frac{S_p \cdot L_p}{D_p}$$

Surface Recombination: Example III (cont'd)

$$p(x) = p_0 + p_g \left\{ 1 - \frac{r \cdot \sinh\left(x/L_p\right) + (1-R) \cdot \left\{\cosh\left[(w-x)/L_p\right] + r \cdot \sinh\left[(w-x)/L_p\right]\right\}}{\cosh\left(w/L_p\right) + r \cdot \sinh\left(w/L_p\right)} \right\}$$

$$J_p(x) = \frac{q \cdot D_p}{L_p} \cdot p_g \cdot \left[\frac{r \cdot \cosh\left(x/L_p\right) - (1-R) \cdot \left\{\sinh\left[(w-x)/L_p\right] + r \cdot \cosh\left[(w-x)/L_p\right]\right\}}{\cosh\left(w/L_p\right) + r \cdot \sinh\left(w/L_p\right)} \right]$$

$$J_p(0) = \frac{q \cdot D_p}{L_p} \cdot p_g \cdot \left\{ \frac{r - (1-R) \cdot \left[\sinh\left(w/L_p\right) + r \cdot \cosh\left(w/L_p\right)\right]}{\cosh\left(w/L_p\right) + r \cdot \sinh\left(w/L_p\right)} \right\}$$

We want $-J_p(0)$ large.

$\rightarrow$ Keep S_p small.