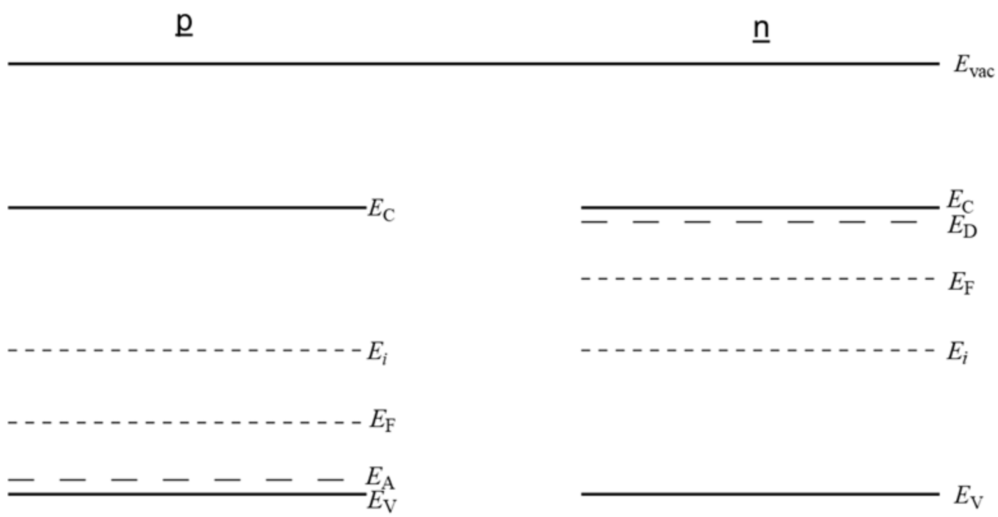
Nanomaterials for Photovoltaics (v11)

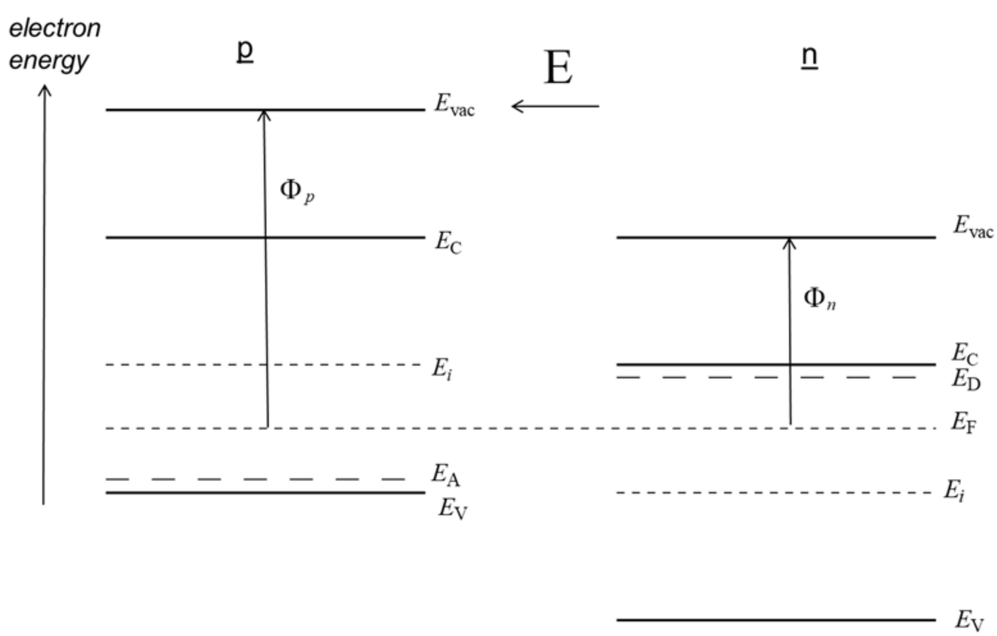
6. Homojunctions

p/n junction diode

The most important device concept for the conversion of light into electrical current is the p/n junction diode. We first consider isolated p and n regions with no electric contact. In this case, we cannot assume the materials to be in electrical equilibrium with one another. In other words, the Fermi level is independent in each material:



In equilibrium, the Fermi level becomes constant by a redistribution of mobile carriers in the vicinity of the junction to form an electric field, with a concomitant band bending. Electrons tend to lowest available energy, so we can infer that the junction has a built-in electric field point from n to p



We can find the built-in potential (voltage) V_{bi} without any detailed knowledge of the carrier distributions near the junction. First find the work function on each side of the junction. On the p-side:

$$\Phi_p = \left(E_{\mathrm{vac}} - E_{\mathrm{F}}\right)\Big|_p = \left(E_{\mathrm{vac}} - E_{\mathrm{C}}\right) + \left(E_{\mathrm{C}} - E_{\mathrm{F}}\right)$$
$$= \chi - \left(E_{\mathrm{F}} - E_{\mathrm{V}}\right) + \left(E_{\mathrm{C}} - E_{\mathrm{V}}\right)$$
$$\Phi_p = \chi - kT \cdot \ln\left(\frac{N_{\mathrm{V}}}{N_{\mathrm{A}}}\right) + E_g$$

On the n-side:

$$\Phi_n = \left(E_{\mathrm{vac}} - E_{\mathrm{F}}\right)\Big|_n = \left(E_{\mathrm{vac}} - E_{\mathrm{C}}\right) + \left(E_{\mathrm{C}} - E_{\mathrm{F}}\right)$$
$$\Phi_n = \chi + kT \cdot \ln\left(\frac{N_C}{N_{\mathrm{D}}}\right)$$

The voltage differences is

$$qV = \Phi_p - \Phi_n = E_{\mathrm{vac}}\big|_p - E_{\mathrm{vac}}\big|_n = \left(\Phi_p + E_{\mathrm{F}}\big|_p\right) - \left(\Phi_n + E_{\mathrm{F}}\big|_n\right)$$

In equilibrium:

$$E_{\mathrm{F}}\big|_p = E_{\mathrm{F}}\big|_n = E_{\mathrm{F}}$$

This gives

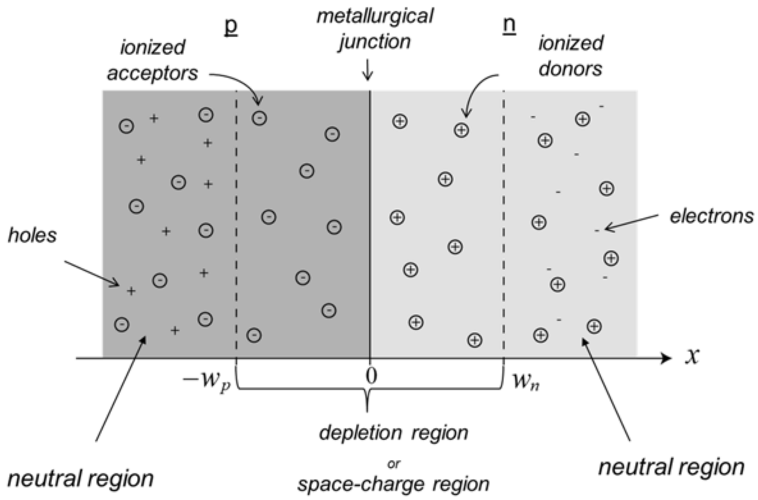
$$qV = \Phi_p - \Phi_n = E_g - kT \cdot \ln\left(\frac{N_{\mathrm{C}} \cdot N_{\mathrm{V}}}{N_{\mathrm{A}} \cdot N_{\mathrm{D}}}\right)$$
$$= E_g - kT \cdot \ln\left(\frac{n_i^{\,2} \cdot \mathrm{e}^{E_g/kT}}{N_{\mathrm{A}} \cdot N_{\mathrm{D}}}\right)$$
$$qV = kT \cdot \ln\left(\frac{N_{\mathrm{A}} \cdot N_{\mathrm{D}}}{n_i^{\,2}}\right)$$

The equilibrium voltage drop across the junction is called the built-in voltage:

$$V_{bi} = \frac{kT}{q} \cdot \ln\left(\frac{N_{\mathrm{A}} \cdot N_{\mathrm{D}}}{n_i^{\,2}}\right)$$

Depletion approximation: space-charge region

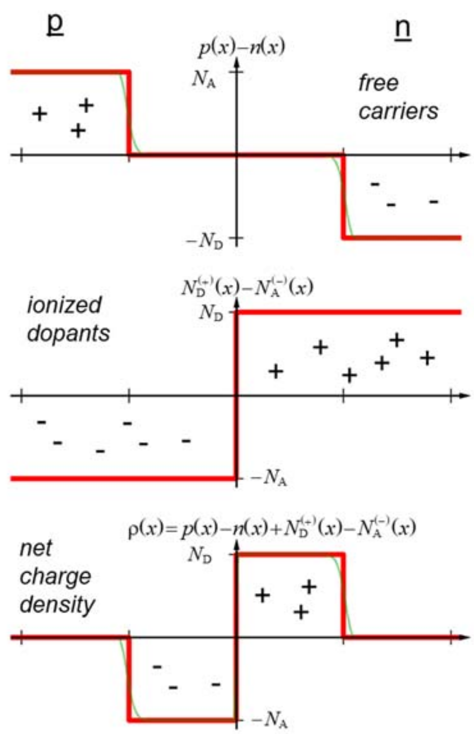
We often need to estimate E near the junction. A useful analysis uses the depletion approximation. It is assume that adjacent regions on either side of the junction are fully depleted of mobile carriers, with only the immobile, ionized dopants contributing to the electric field. The problem is then one of electrostatics. This charged volume near the junction is called the *depletion region* or the *space-charge region* (SCR).



Depletion approximation: charge density
We are left with ionized acceptors and donors in the SCR. Within this approximation, the charge density is given by:

$$\rho(x)=\begin{cases} 0, & x<-w_p \\ -N_A, & -w_p \le x<0 \\ +N_D, & 0\le x<w_n \\ 0, & w_n \le x \end{cases}$$

The unknown parameters here are the widths w_p and w_n.



Depletion approximation: electric field
The Poisson equation relates the charge density to the electric field.

$$\frac{d\mathrm{E}}{dx} = \frac{\rho(x)}{\varepsilon}$$

Overall charge neutrality dictates that the field must vanish far from the junction. Then

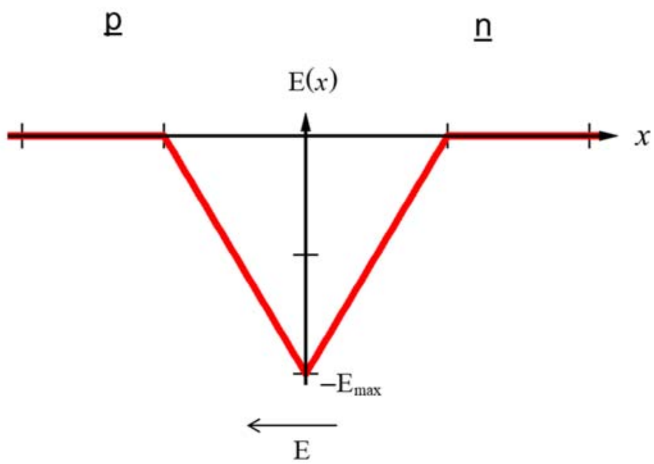
$$\mathrm{E} = \int_x \left[\frac{q \cdot \rho(x)}{\varepsilon}\right] \cdot dx = \begin{cases} 0, & x < -w_p \\ -\frac{qN_\mathrm{A}}{\varepsilon} \cdot (x + w_p), & -w_p \le x < 0 \\ -\frac{qN_\mathrm{D}}{\varepsilon} \cdot (w_n - x), & 0 \le x < w_n \\ 0, & w_n \le x \end{cases}$$

Assuming no surface charges are present at the junction, the field must be continuous there:

$$\mathrm{E}|_{x=0^-} = \mathrm{E}|_{x=0^+},$$

which requires that $N_\mathrm{A} \cdot w_p = N_\mathrm{D} \cdot w_n$. The maximum field is then

$$\mathrm{E}_{\max} = \frac{qN_\mathrm{A} \cdot w_p}{\varepsilon} = \frac{qN_\mathrm{D} \cdot w_n}{\varepsilon}$$

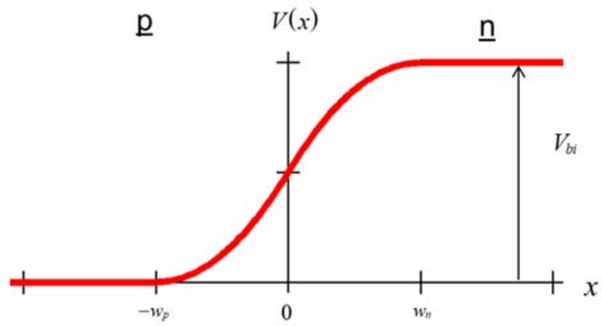


Depletion approximation: potential
The electric field is the gradient of the potential, or $\mathrm{E} = -dV/dx$ in 1-D. Integrating

$$V(x) = \int_{x'} \mathrm{E}(x') \cdot dx' = \begin{cases} 0, & x < -w_p \\ \frac{qN_\mathrm{A}}{2\varepsilon} \cdot (x + w_p)^2, & -w_p \le x < 0 \\ \frac{q}{2\varepsilon} \cdot \left\{N_\mathrm{A} \cdot w_p^2 + N_\mathrm{D} \cdot \left[w_n^2 - (w_n - x)^2\right]\right\}, & 0 \le x < w_n \\ \frac{q}{2\varepsilon} \cdot \left(N_\mathrm{A} \cdot w_p^2 + N_\mathrm{D} \cdot w_n^2\right), & w_n \le x \end{cases}$$

We find that the built-in potential can be written as

$$V_{bi} = \frac{q}{2\varepsilon} \cdot \left(N_A \cdot w_p^{\;2} + N_D \cdot w_n^{\;2} \right)$$



Depletion approximation: depletion width
We know that

$$w_p = \frac{N_D}{N_A} \cdot w_n \,, \; w_n = \frac{N_A}{N_D} \cdot w_p$$

We can write the built-in potential in terms of either w_p or w_n:

$$V_{bi} = \frac{q}{2\varepsilon} \cdot \left(\frac{1}{N_A} + \frac{1}{N_D} \right) \cdot \left(N_A \cdot w_p \right)^2 = \frac{q}{2\varepsilon} \cdot \left(\frac{1}{N_A} + \frac{1}{N_D} \right) \cdot \left(N_D \cdot w_n \right)^2$$

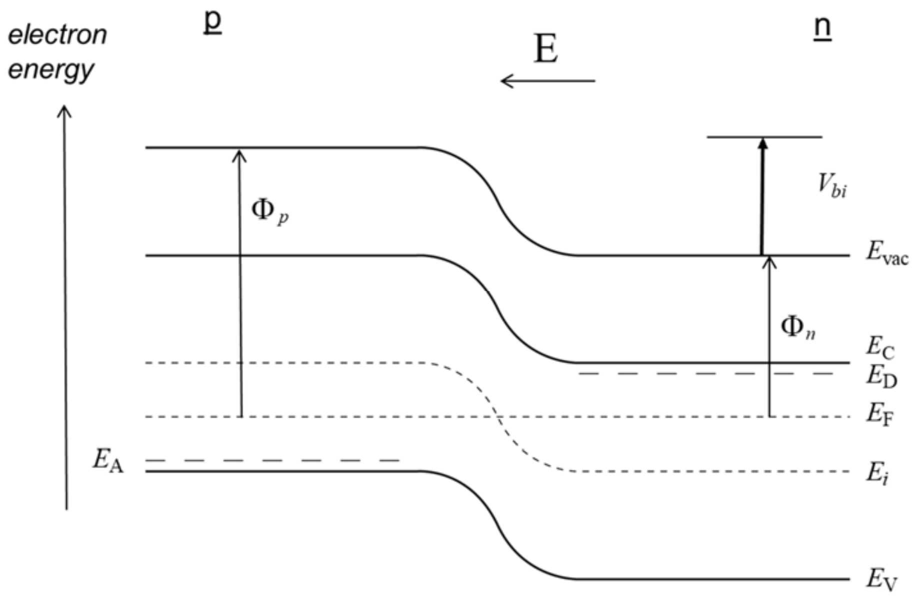
As we already know V_{bi} from energy considerations, we can then solve for the dimensions of the space-charge region:

$$w_p = \frac{1}{N_A} \cdot \sqrt{\frac{2\varepsilon V_{bi}}{q} \cdot \left(\frac{1}{\frac{1}{N_A} + \frac{1}{N_D}} \right)} \,, \; w_n = \frac{1}{N_D} \cdot \sqrt{\frac{2\varepsilon V_{bi}}{q} \cdot \left(\frac{1}{\frac{1}{N_A} + \frac{1}{N_D}} \right)}$$

The total depletion width is then:

$$w = w_p + w_n = \sqrt{\frac{2\varepsilon V_{bi}}{q} \cdot \left(\frac{1}{N_A} + \frac{1}{N_D} \right)}$$

Depletion approximation: energy diagram
Finally, we can plot the complete band diagram within the depletion approximation for the p/n junction in equilibrium.



p/n junction: non-equilibrium (I)
Let's include the influence of an external voltage V , typically, which may be due to an applied bias or to photogeneration. The voltage drop across the SCR is apparently

$$\int_{x=-w_p}^{w_n} \mathrm{E}(x)\cdot dx = \frac{q}{2\varepsilon}\left(N_{\mathrm{A}}\cdot w_p^{\,2} + N_{\mathrm{D}}\cdot w_n^{\,2}\right) = V_{bi} - V$$

Using the previous derivation, the width of the SCR is:

$$w = w_p + w_n = \sqrt{\frac{2\varepsilon\cdot\left(V_{bi}-V\right)}{q}\cdot\left(\frac{1}{N_{\mathrm{A}}}+\frac{1}{N_{\mathrm{D}}}\right)}$$

Notice that the SCR becomes narrower in forward bias ($V > 0$) and wider in reverse bias ($V < 0$). In the neutral regions (outside of the SCR),we expect

$$p = N_{\mathrm{A}} = n_i\cdot \mathrm{e}^{\left(E_i(-\infty)-E_{\mathrm{F}_p}(-\infty)\right)/kT},\; n = N_{\mathrm{D}} = n_i\cdot \mathrm{e}^{\left(E_{\mathrm{F}_n}(\infty)-E_i(\infty)\right)/kT}$$

In the SCR, the quasi-Fermi levels are approximately determined by those of the majority carrier on either side. (We will look at this more carefully later).

$$E_{\mathrm{F}_p} \approx E_{\mathrm{F}_p}(-\infty),\; E_{\mathrm{F}_n} \approx E_{\mathrm{F}_n}(\infty)$$

Consider the product

$$N_{\mathrm{A}}\cdot N_{\mathrm{D}} = n_i^{\,2}\cdot \mathrm{e}^{\left[\left(E_{\mathrm{F}_n}-E_{\mathrm{F}_p}\right)-\left(E_i(\infty)-E_i(-\infty)\right)\right]/kT} = n_i^{\,2}\cdot \mathrm{e}^{\left[\Delta\mu + q\cdot\left(V_{bi}-V\right)\right]/kT}$$

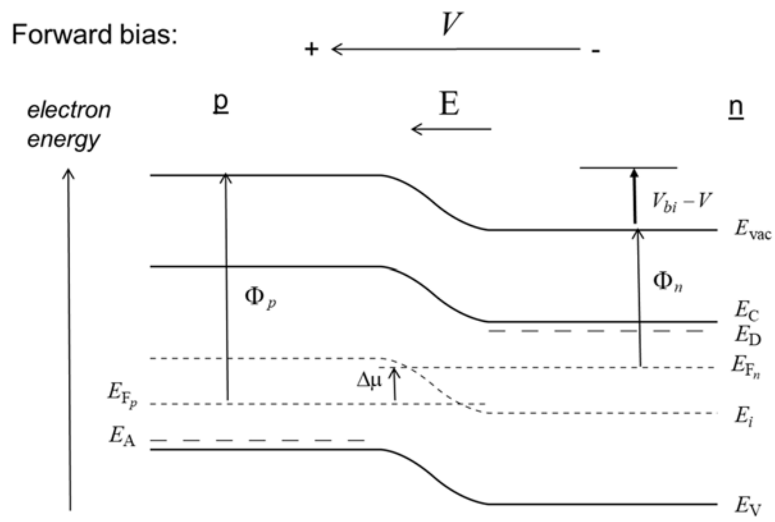
Let's find the quasi-Fermi level splitting in the SCR:

$$\Delta\mu = E_{\mathrm{F}_n}(\infty) - E_{\mathrm{F}_p}(-\infty) = q\left(V - \cancel{V_{bi}}\right) + kT\cdot\ln\left(\cancel{\frac{N_{\mathrm{A}}\cdot N_{\mathrm{D}}}{n_i^{2}}}\right) = qV$$

This shows that the quasi-Fermi level splitting in the SCR is approximately *equal* to the external voltage.

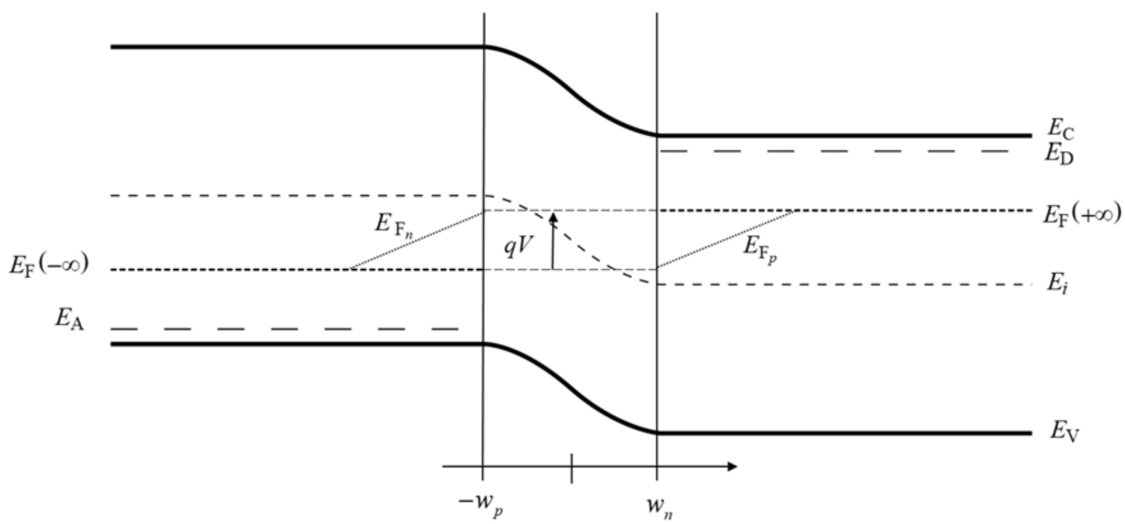
p/n junction: non-equilibrium (II)

Based on the preceding discussion, we can sketch the energy diagram for the p/n junction in forward bias ($V > 0$) as shown below:



p/n junction: non-equilibrium (III)

We can sketch a slightly more complete energy diagram for a p/n junction in forward bias ($V > 0$). If there are no photogenerated carriers there is no quasi-Fermi level splitting outside the SCR:



It is fairly easy to see that the Fermi level far from the junction is approximately determined by the majority carrier concentration.

p/n junction: non-equilibrium (IV)

First, analyze without photogeneration. Far from the junction:

$$n(-\infty) = n_0^{(p)} = \frac{n_i^2}{N_a}, \quad p(\infty) = p_0^{(n)} = \frac{n_i^2}{N_d}$$

In the SCR:

$$n \cdot p = n_i^2 \cdot e^{\Delta\mu/kT}$$

where $\Delta\mu = qV$. The depletion approximation gives:

$$p(-w_p) = N_a \qquad n(w_n) = N_d$$
$$n(-w_p) = n_0^{(p)} \cdot e^{qV/kT}, \quad p(w_n) = p_0^{(n)} \cdot e^{qV/kT}$$

The excess minority-carrier concentrations at the edges of the SCR are

$$\Delta n(-w_p) = n(-w_p) - n_0^{(p)} = n_0^{(p)} \cdot (e^{qV/kT} - 1)$$
$$\Delta p(w_n) = p(w_n) - p_0^{(n)} = p_0^{(n)} \cdot (e^{qV/kT} - 1)$$

These are critical for determining the photocurrent.

Photogeneration rate

Absorption

$$\frac{d}{dx}[b(E,x)] = -\alpha(E,x) \cdot b(E,x)$$

where $\alpha(E,x)$ is the local absorption coefficient and $b(E,x)$ is the spectral photon flux density in the material. We can treat each energy separately. Let's assume uniform absorption [$\alpha(E,x) = \alpha(E)$]. Separating variables

$$\int_{b'=b(0)}^{b(x)} \frac{db'}{b'} = -\int_{x'=0}^{x} \alpha(x') \cdot dx' \to -\alpha x$$
$$\ln\left[\frac{b(x)}{b(0)}\right] = -\alpha x$$

This gives

$$b(E,x) = b(E,0) \cdot e^{-\alpha(E) \cdot x}$$

Note that

$$b(E,0) = [1 - R(E)] \cdot b_S(E)$$

Carrier generation rate is

$$g(E,x) = -\frac{d}{dx} b(E,x)$$
$$= \alpha(E) \cdot b(E,x)$$
$$g(E,x) = [1 - R(E)] \cdot \alpha(E) \cdot b_S(E) \cdot e^{-\alpha(E) \cdot x}$$

p/n junction solar cell (I)

Let's assume weak absorption, so that there is no attenuation of the incident illumination. Then $e^{-\alpha(E) \cdot x} \approx 1$. The photogeneration rate is

$$g(E,x) \approx [1 - R(E)] \cdot \alpha(E) \cdot b_s(E) \cdot (1) \to g(E)$$

This gives uniform generation, so that

$$G(x) \approx \int_{E=0}^{\infty} dE \cdot g(E) \to G$$

If this is band-to-band absorption, then $G_n = G_p = G$. In steady state, on the p-side:

$$n(-\infty) \to n^{(p)} = n_0^{(p)} + n_g$$

$$G_n = U_n = \frac{n^{(p)} - n_0^{(p)}}{\tau_n} = \frac{n_g}{\tau_n} = G$$

So, $n_g = G \cdot \tau_n$. On the n-side

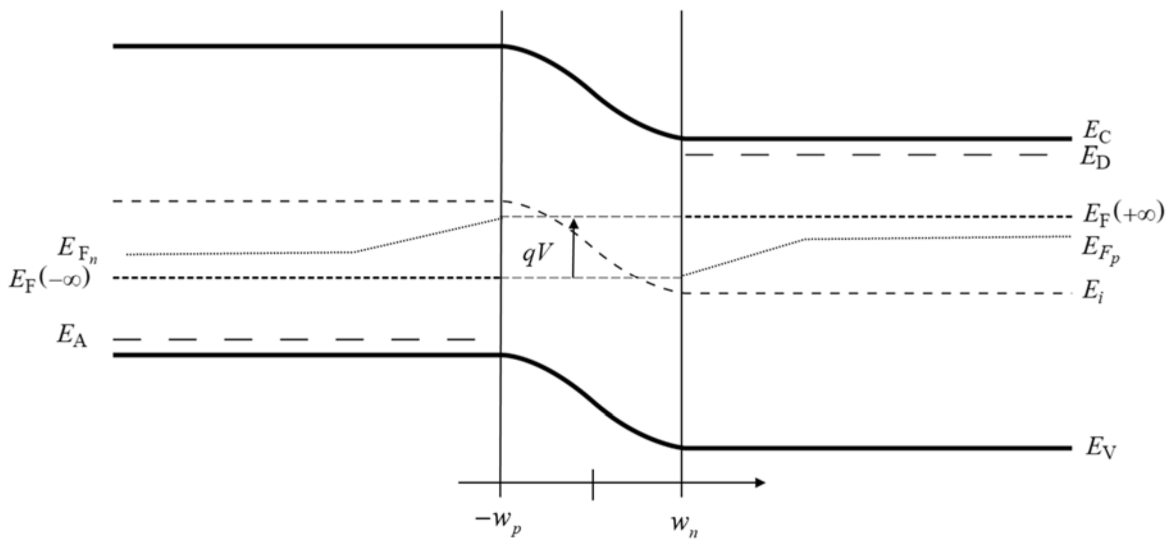
$$p(\infty) \to p^{(n)} = p_0^{(n)} + p_g$$

$$G_p = U_p = \frac{p^{(n)} - p_0^{(n)}}{\tau_p} = \frac{p_g}{\tau_p} = G$$

which gives $p_g = G \cdot \tau_p$.

p/n junction solar cell (II)

With uniform photogeneration, there are excess minority carriers in the neutral regions, so the quasi-Fermi levels are split:



With moderate photogeneration, the majority carrier densities are not largely affected, but the minority carrier densities are.

p/n junction solar cell (III)

Consider generation in the quasi-neutral regions (assuming uniform generation). On the p (n) side, outside the SCR

$$\Delta n(x) = n(x) - n^{(p)}, \ \Delta p(x) = p(x) - p^{(n)}$$

At the edges of the SCR

$$\Delta n(-w_p) = n(-w_p) - n^{(p)}, \quad \Delta p(w_n) = p(w_n) - p^{(n)}$$

The carrier concentrations are still related to the quasi Fermi-level spitting.

$$n(-w_p) = n_0^{(p)} \cdot e^{qV/kT}, \qquad p(w_n) = p_0^{(n)} \cdot e^{qV/kT}$$

So the excess minority-carrier concentration at the edges of the SCR are:

$$\Delta n(-w_p) = n_0^{(p)} \cdot e^{qV/kT} - \left(n_0^{(p)} + n_g\right) = n_0^{(p)} \cdot \left(e^{qV/kT} - 1\right) - n_g$$

$$\Delta p(w_n) = p_0^{(n)} \cdot e^{qV/kT} - \left(p_0^{(n)} + p_g\right) = p_0^{(n)} \cdot \left(e^{qV/kT} - 1\right) - p_g$$

p/n junction solar cell (IV)
The total current density is

$$J = -J_n(x) - J_p(x)$$

where the sign convention used for PV is adopted, such that J is positive when the device is delivering power. This current must be constant at every point in the circuit, so

$$J = -J_n(-w_p) - J_p(-w_p) = -J_n(w_n) - J_p(w_n)$$

The depletion approximation implies that all the potential difference occurs across the SCR. So, E=0 at the SCR edges ($x = -w_p,\ x = w_n$). At these points:

$$J_n = q \cdot \mu_n \cdot E + q \cdot D_n \cdot \frac{dn}{dx} \to q \cdot D_n \cdot \frac{dn}{dx}$$

$$J_p = q \cdot \mu_p \cdot E - q \cdot D_p \cdot \frac{dp}{dx} \to -q \cdot D_p \cdot \frac{dp}{dx}$$

p/n junction solar cell (V)
In the neutral regions, we have diffusion only:

$$\Delta n(x) = A_n \cdot \mathrm{e}^{-(x+w_p)/L_n} + B_n \cdot \mathrm{e}^{(x+w_p)/L_n},\ x < -w_p$$

$$\Delta p(x) = A_p \cdot \mathrm{e}^{-(x-w_n)/L_p} + B_p \cdot \mathrm{e}^{(x-w_p)/L_p},\ w_n < x$$

Assume infinitely thick neutral regions, $\Delta n(-\infty) = 0$ and $\Delta p(\infty) = 0$. The boundary conditions give

$$\Delta n(x) = \Delta n(-w_p) \cdot \mathrm{e}^{(x+w_p)/L_n}, \qquad x < -w_p$$

$$\Delta p(x) = \Delta p(w_n) \cdot \mathrm{e}^{-(x-w_n)/L_p},\ w_n < x$$

The currents are

$$J_n(x) = q \cdot D_n \cdot \frac{d(\Delta n)}{dx} = \frac{q \cdot D_n}{L_n} \cdot \Delta n(x)$$

$$J_p(x) = -q \cdot D_p \cdot \frac{d(\Delta p)}{dx} = \frac{q \cdot D_p}{L_p} \cdot \Delta p(x)$$

So we know that

$$J_n(-w_p) = \frac{q \cdot D_n}{L_n} \cdot \Delta n(-w_p) \text{ and } J_p(w_n) = \frac{q \cdot D_p}{L_p} \cdot \Delta p(w_n)$$

From previous results

$$\Delta n(-w_p) = n_0^{(p)} \cdot (\mathrm{e}^{qV/kT} - 1) - n_g$$

$$\Delta p(w_n) = p_0^{(n)} \cdot (\mathrm{e}^{qV/kT} - 1) - p_g$$

So

$$J_n(-w_p) = \frac{q \cdot D_n}{L_n} \cdot [n_0^{(p)} \cdot (\mathrm{e}^{qV/kT} - 1) - n_g]$$

$$J_p(w_n) = \frac{q \cdot D_p}{L_p} \cdot [p_0^{(n)} \cdot (\mathrm{e}^{qV/kT} - 1) - p_g]$$

p/n junction solar cell (VI)
We know $J_n(-w_p)$ and $J_p(w_n)$. To find

$$J = -J_n(-w_p) - J_p(-w_p) = -J_n(w_n) - J_p(w_n)$$

we need either

$$J_p(-w_p) = J_p(w_n) + J_{p,\mathrm{SCR}}(-w_p)$$

or

$$J_n(w_n) = J_n(-w_p) + J_{n,\mathrm{SCR}}(w_n)$$

where

$$J_{p,\mathrm{SCR}}(-w_p) = q \cdot \int_{x=-w_p}^{w_n} [U(x) - G(x)] \cdot dx$$

$$= J_{p,\mathrm{rec}} - J_{p,\mathrm{gen}}$$

and

$$J_{n,\mathrm{SCR}}(w_n) = q \cdot \int_{x=-w_p}^{w_n} [U(x) - G(x)] \cdot dx$$

$$= J_{n,\mathrm{rec}} - J_{n,\mathrm{gen}}$$

p/n junction solar cell (VII)

Let's ignore recombination in the SCR, so $J_{p,rec} \approx J_{n,rec} \approx 0$. If there is uniform generation in the SCR

$$J_{n,\mathrm{gen}} = J_{p,\mathrm{gen}} = q \cdot \int_{x=-w_p}^{w_n} G \cdot dx = q \cdot G \cdot w$$

So, we can say that

$$J_{p,\mathrm{SCR}}(-w_p) = J_{n,\mathrm{SCR}}(w_n) \approx -q \cdot G \cdot w$$

We just need one or the other. Let's find $J = -J_p(w_n) - J_n(w_n)$. We know

$$J_p(w_n) = \frac{q \cdot D_p}{L_p} \cdot \left[p_0^{(n)} \cdot (\mathrm{e}^{qV/kT} - 1) - p_g \right]$$

$$J_n(w_n) = J_n(-w_p) + J_{n,SCR}(w_n)$$

where

$$J_n(-w_p) = \frac{q \cdot D_n}{L_n} \cdot \left[n_0^{(p)} \cdot (\mathrm{e}^{qV/kT} - 1) - n_g \right]$$

and

$$J_{n,\mathrm{SCR}}(w_n) \approx -q \cdot G \cdot w$$

Putting these together

$$J_n(w_n) = J_n(-w_p) + J_{n,SCR}(w_n) \approx -q \cdot G \cdot w = \frac{q \cdot D_n}{L_n} \cdot \left[n_0^{(p)} \cdot (\mathrm{e}^{qV/kT} - 1) - n_g \right] - q \cdot G \cdot w$$

p/n junction solar cell (VIII)

We need

$$J = -J_p(w_n) - J_n(w_n)$$

We have

$$J_p(w_n) = \frac{q \cdot D_p}{L_p} \cdot \left[p_0^{(n)} \cdot (\mathrm{e}^{qV/kT} - 1) - p_g \right]$$

and

$$J_n(w_n) = -q \cdot G \cdot w = \frac{q \cdot D_n}{L_n} \cdot \left[n_0^{(p)} \cdot (\mathrm{e}^{qV/kT} - 1) - n_g \right] - q \cdot G \cdot w$$

We can write

$$\frac{q \cdot D_p}{L_p} \cdot p_g = \frac{q \cdot D_p}{L_n} \cdot (G \cdot \tau_p) = q \cdot G \cdot L_p \text{ and } \frac{q \cdot D_n}{L_n} \cdot n_g = \frac{q \cdot D_n}{L_n} \cdot (G \cdot \tau_n) = q \cdot G \cdot L_n$$

Then

$$J_p(w_n) = \frac{q \cdot D_p}{L_p} \cdot p_0^{(n)} \cdot (\mathrm{e}^{qV/kT} - 1) - q \cdot G \cdot L_p$$

and

$$J_n(w_n) = \frac{q \cdot D_n}{L_n} \cdot n_0^{(p)} \cdot (\mathrm{e}^{qV/kT} - 1) - q \cdot G \cdot (L_n + w)$$

p/n junction solar cell (IX)

The total current density is

$$J = q \cdot G \cdot (w + L_n + L_p) - \left(\frac{q \cdot D_n}{L_n} \cdot n_0^{(p)} + \frac{q \cdot D_p}{L_p} \cdot p_0^{(n)} \right) \cdot (\mathrm{e}^{qV/kT} - 1)$$

or

$$J = J_{\text{photo}} - J_0 \cdot (\mathrm{e}^{qV/kT} - 1)$$

where the photocurrent is

$$J_{\text{photo}} = q \cdot G \cdot (w + L_n + L_p)$$

The dark (diode) current is

$$J_0 = q \cdot \left(\frac{D_n}{L_n} \cdot n_0^{(p)} + \frac{D_p}{L_p} \cdot p_0^{(n)} \right) = q \cdot n_i^2 \cdot \left(\frac{D_n}{L_n \cdot N_{\mathrm{A}}} + \frac{D_p}{L_p \cdot N_{\mathrm{D}}} \right)$$

Finally, we have the most common form for the p/n junction solar-cell current:

$$J(V) = J_{\text{photo}} - J_0 \cdot (\mathrm{e}^{qV/kT} - 1)$$

SCR transit time

It is difficult to estimate the amount of recombination occurring in the SCR. The carriers move quickly through the SCR, because of the large electric field there. Let’s estimate the transit time for carriers to cross the SCR. For this, we compute the drift velocity in a constant electric field given by the average field in the SCR, which is $\overline{\mathrm{E}} = \mathrm{E}_{\max}/2$ in the depletion approximation. Using $\overline{v} = \mu \overline{\mathrm{E}}$, we have a transit time of

$$\tau = \frac{w}{\overline{v}} = \frac{2w}{\mu \mathrm{E}_{\max}}$$

The maximum field is

$$\mathrm{E}_{\max} = \frac{qN_{\mathrm{A}} \cdot w_p}{\varepsilon} = \frac{qN_{\mathrm{D}} \cdot w_n}{\varepsilon}$$

Using $w = w_p + w_n$

$$N_{\mathrm{A}} \cdot w_p = \frac{1}{\frac{1}{N_{\mathrm{A}}} + \frac{1}{N_{\mathrm{D}}}} \cdot w$$

This gives

$$\mathrm{E}_{\max} = \frac{q}{\varepsilon} \cdot \frac{1}{\frac{1}{N_{\mathrm{A}}} + \frac{1}{N_{\mathrm{D}}}} \cdot w$$

We need the ratio

$$\frac{\mathrm{E}_{\max}}{w} = \frac{q}{\varepsilon} \cdot \frac{1}{\frac{1}{N_{\mathrm{A}}} + \frac{1}{N_{\mathrm{D}}}}$$

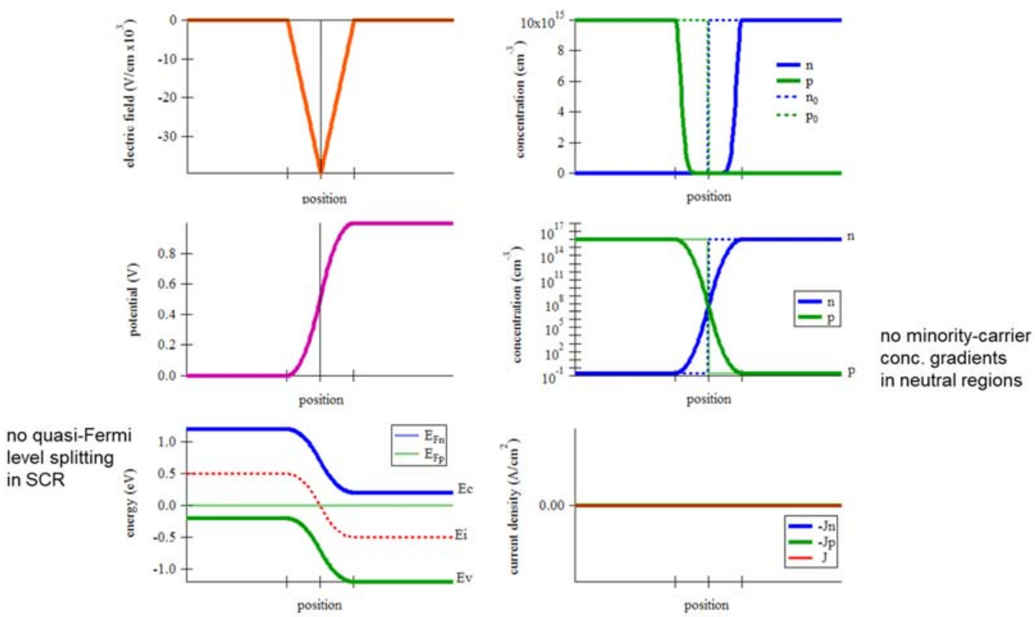
Finally

$$\tau = \frac{2\varepsilon}{\mu q} \cdot \left(\frac{1}{N_{\mathrm{A}}} + \frac{1}{N_{\mathrm{D}}} \right)$$

We can put in parameters for a numerical estimation. Assuming $\varepsilon = 10 \times \varepsilon_0$, $\mu = 100\ \mathrm{cm}^2/\mathrm{V} \cdot \mathrm{s}$, and $N_{\mathrm{A}} = N_{\mathrm{D}} = 10^{16}\ \mathrm{cm}^{-3}$, we get $\tau = 2.2 \times 10^{-11}\ \mathrm{s}$, which is short compared to typical minority carrier lifetimes in high-quality material, so the approximation is reasonably valid, but should be used with appropriate discretion.

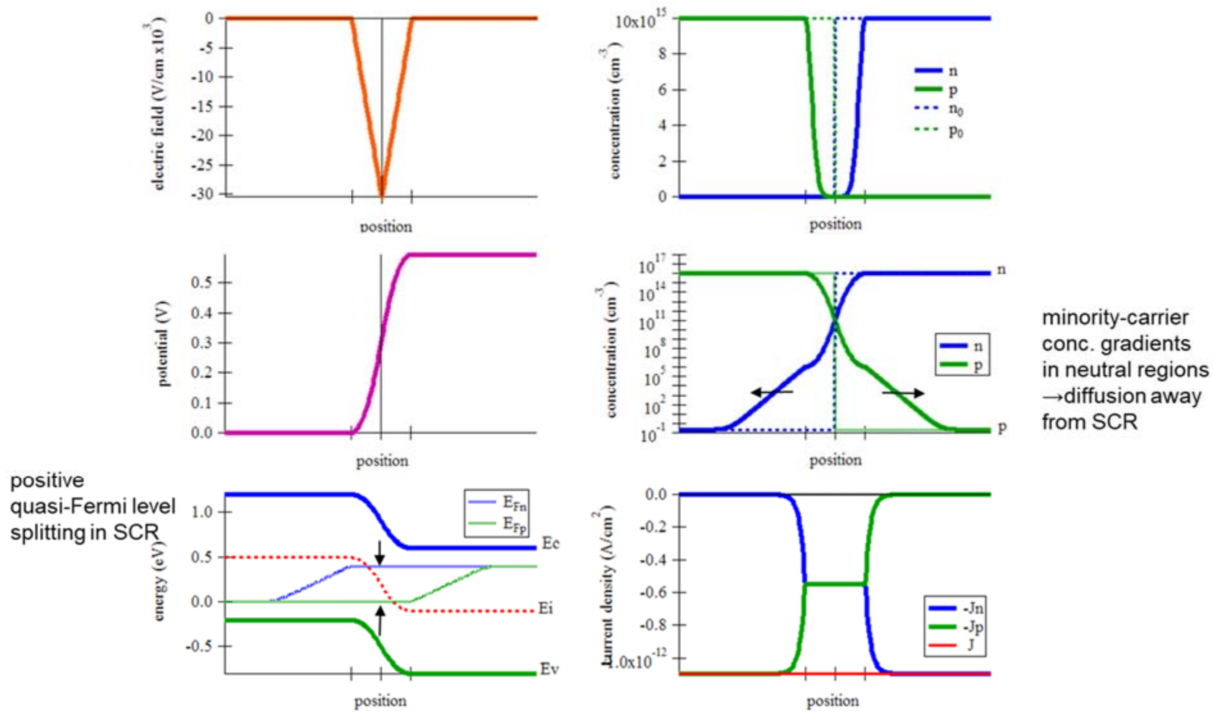
Plots: depletion approx. (I)

Case 1: equilibrium ($V = 0\ \mathrm{V},\ G = 0$)



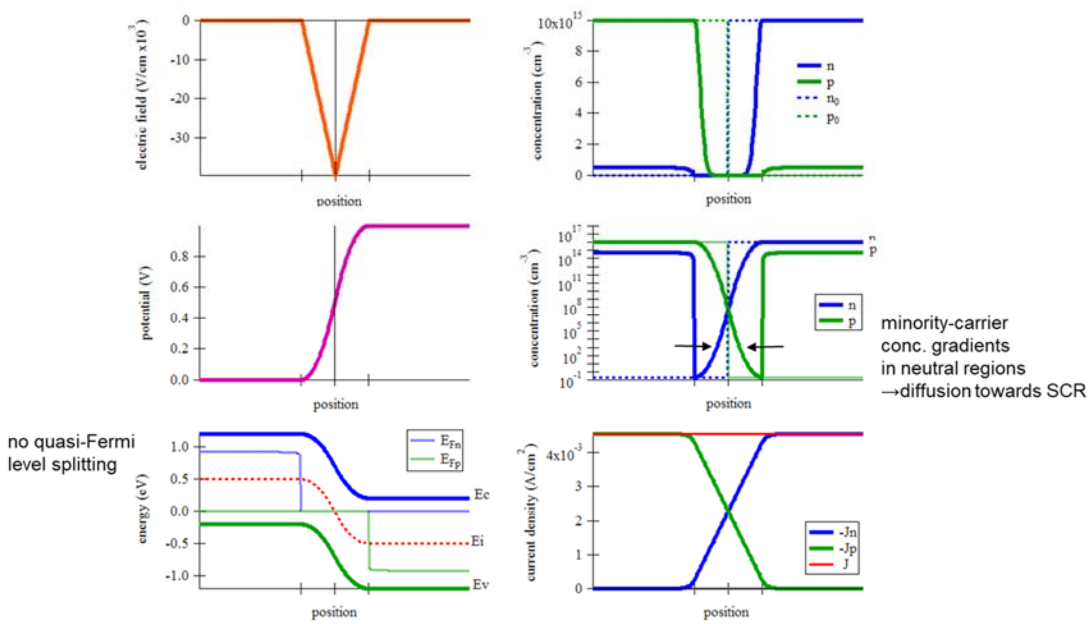
Plots: depletion approx. (II)

Case 2: forward bias, no illumination ($V = 0.5\ \mathrm{V},\ G = 0$)



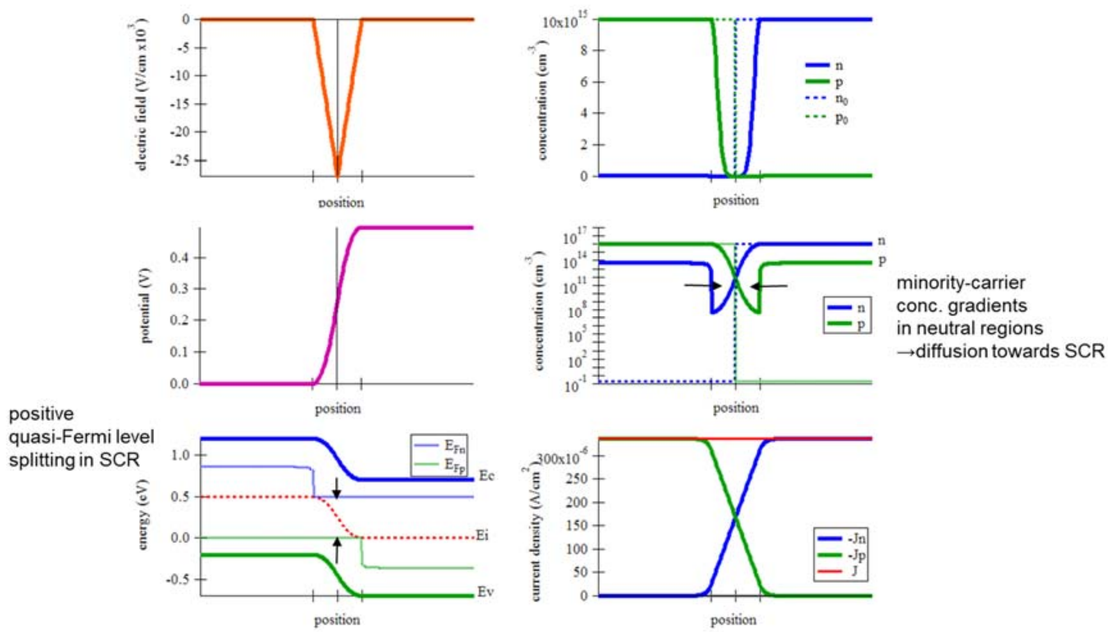
Plots: depletion approx. (III)

<u>Case 3</u>: short circuit, illuminated ($V = 0$ V, $G = 5 \times 10^{19}$ $1/(\mathrm{cm}^3 \cdot \mathrm{s})$)



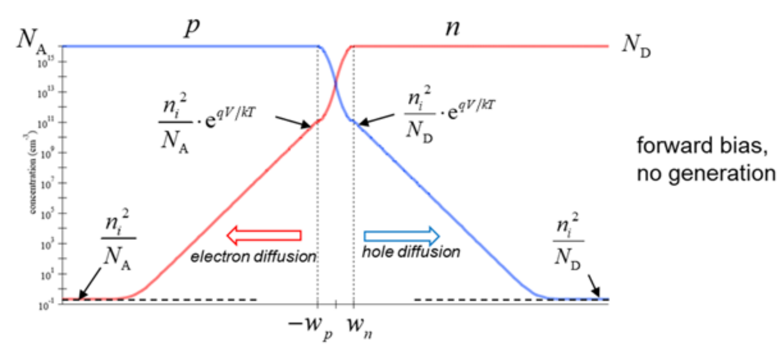
Plots: depletion approx. (IV)

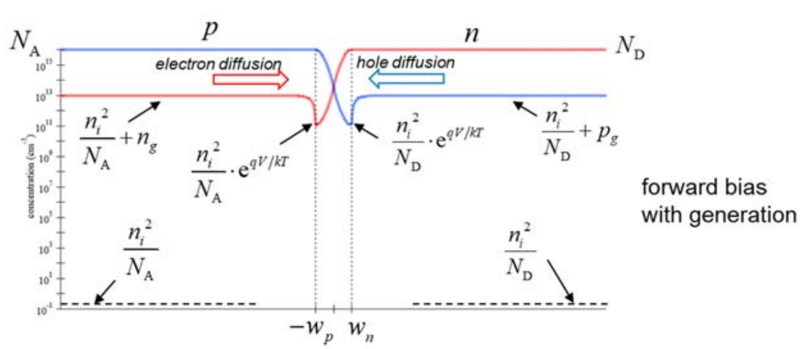
<u>Case 4</u>: forward bias, illuminated ($V = 0.5$ V, $G = 5 \times 10^{19}$ $1/(\mathrm{cm}^3 \cdot \mathrm{s})$)



Plots: carrier concentrations in depletion approx.

The plots below show the mechanisms for current in the cases of forward bias with no generation and forward bias with generation. In the depletion approximation, the minority-carrier concentrations at the edges of the SCR are independent of their concentrations farther from the junction. So, if no photogeneration occurs, minority carriers will diffusive away from the junction, which constitutes the diode current. If adequate photogeneration occurs, minority carriers will diffuse towards the junction and contribute to the photocurrent.





Analysis without the depletion approx.

In the following simulations, we have assumed the primary recombination mechanism is radiative, by specifying long SRH lifetimes. Dashed curves were computed using the depletion approximation. Solid curves were computed by varying the quasi-Fermi levels to satisfy the underlying equations:

1) Poisson equation

$$\frac{d^2V}{dx^2} = -\frac{q\cdot\rho(x)}{\varepsilon}$$

2) electron and hole current densities

$$J_n = \mu_n \cdot n \cdot \frac{dE_{F_n}}{dx},\ J_p = \mu_p \cdot p \cdot \frac{dE_{F_p}}{dx}$$

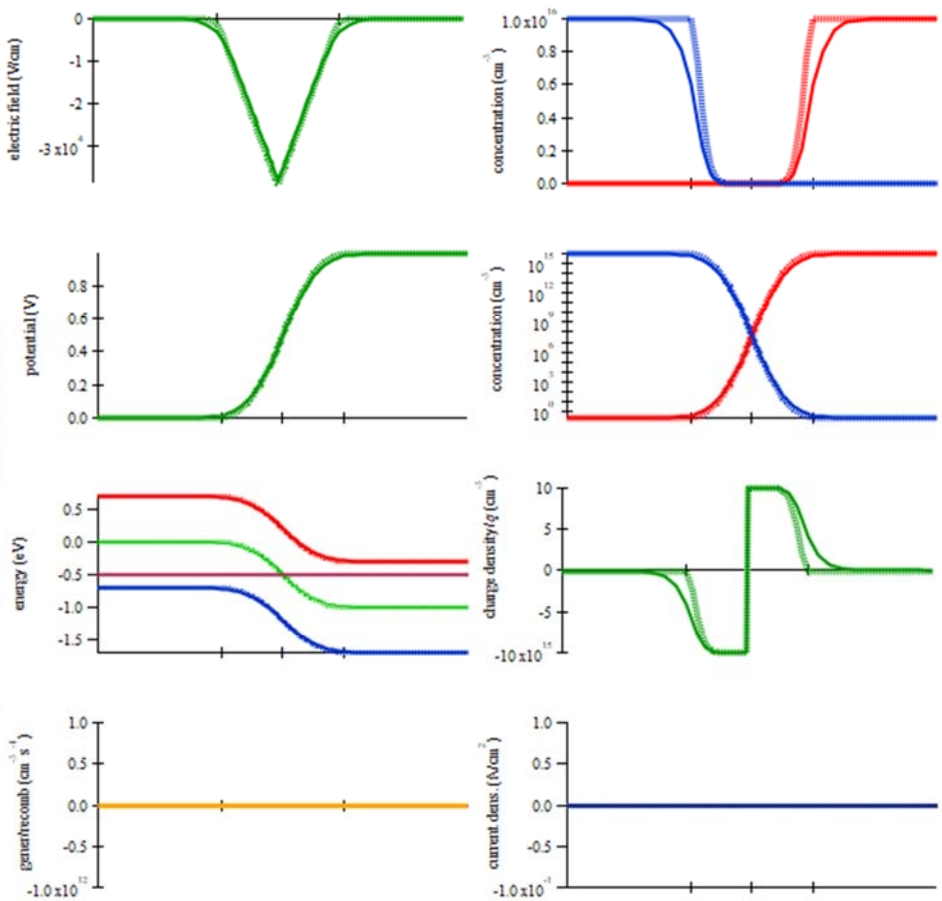
3) electron and hole continuity

$$-\frac{1}{q}\frac{dJ_n}{dx} = G_n - U_n,\ \frac{1}{q}\frac{dJ_p}{dx} = G_p - U_p$$

Plots: no depletion approx. (I)

inputLabel	inputVal	inputUnit
Na	1e+16	cm^-3
Nd	1e+16	cm^-3
mc	1	
mv	1	
kT	0.0259	eV
Eg	1.4	eV
mu_n	10	cm^2/V/s
mu_p	10	cm^2/V/s
eps_s	11.6	
G	0	1/cm^3/s
Va	0	V
Brad	1e-06	cm^3/s
tau_nSRH	1	s
tau_pSRH	1	s
dEt	0	eV

outputLabel	outputVal	outputUnit
Vbi	0.994395	V
J0	0.0018877	A
J0dark	3.45338e-15	A



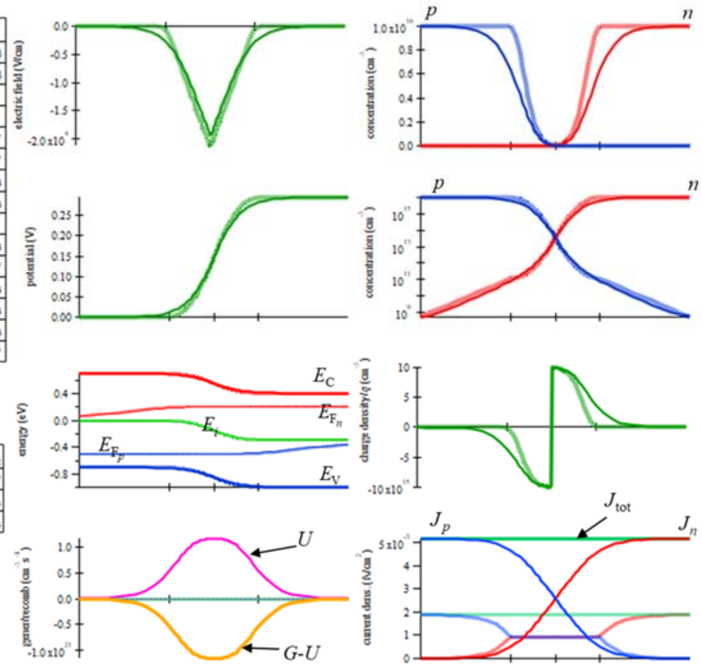
Plots: no depletion approx. (I)

We used the depletion approximation to estimate the dimensions of the SCR, from which the rest of the p/n junction properties were derived. We assumed no recombination in the SCR. These assumptions provide a general insight into the J-V characteristics of the device. It is possible to find more accurate descriptions of these characteristics without so many assumptions.

<u>Case 1</u>: forward bias, no generation ($V = 0.7$ V, $G = 0$)

inputLabel	inputVal	inputUnit
Na	1e+16	cm^-3
Nd	1e+16	cm^-3
mc	1	
mv	1	
kT	0.0259	eV
Eg	1.4	eV
mu_n	10	cm^2/V/s
mu_p	10	cm^2/V/s
eps_s	11.6	
G	0	1/cm^3/s
Va	0.7	V
Brad	1e-06	cm^3/s
tau_nSRH	1	s
tau_pSRH	1	s
dEt	0	eV

outputLabel	outputVal	outputUnit
Vbi	0.994395	V
J0	0.0018877	A
J0dark	3.45338e-15	A



Plots: no depletion approx. (II)

<u>Case 2</u>: operating, illuminated ($V = 0.7$ V, $G = 1 \times 10^{23}$ $1/(\mathrm{cm}^3 \cdot \mathrm{s})$)

inputLabel	inputVal	inputUnit
Na	1e+16	cm^-3
Nd	1e+16	cm^-3
mc	1	
mv	1	
kT	0.0259	eV
Eg	1.4	eV
mu_n	10	cm^2/V/s
mu_p	10	cm^2/V/s
eps_s	11.6	
G	1e+23	1/cm^3/s
Va	0.7	V
Brad	1e-06	cm^3/s
tau_nSRH	1	s
tau_pSRH	1	s
dEt	0	eV

outputLabel	outputVal	outputUnit
Vbi	0.994395	V
J0	-0.764468	A
J0dark	3.45338e-15	A

depl. approx.: dashed lines
full calc.: solid lines