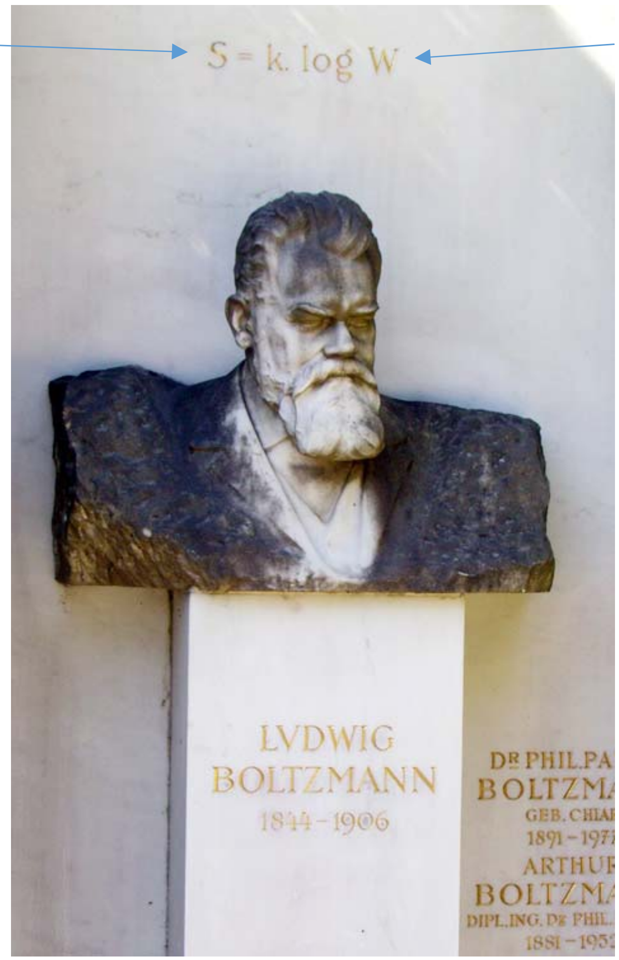
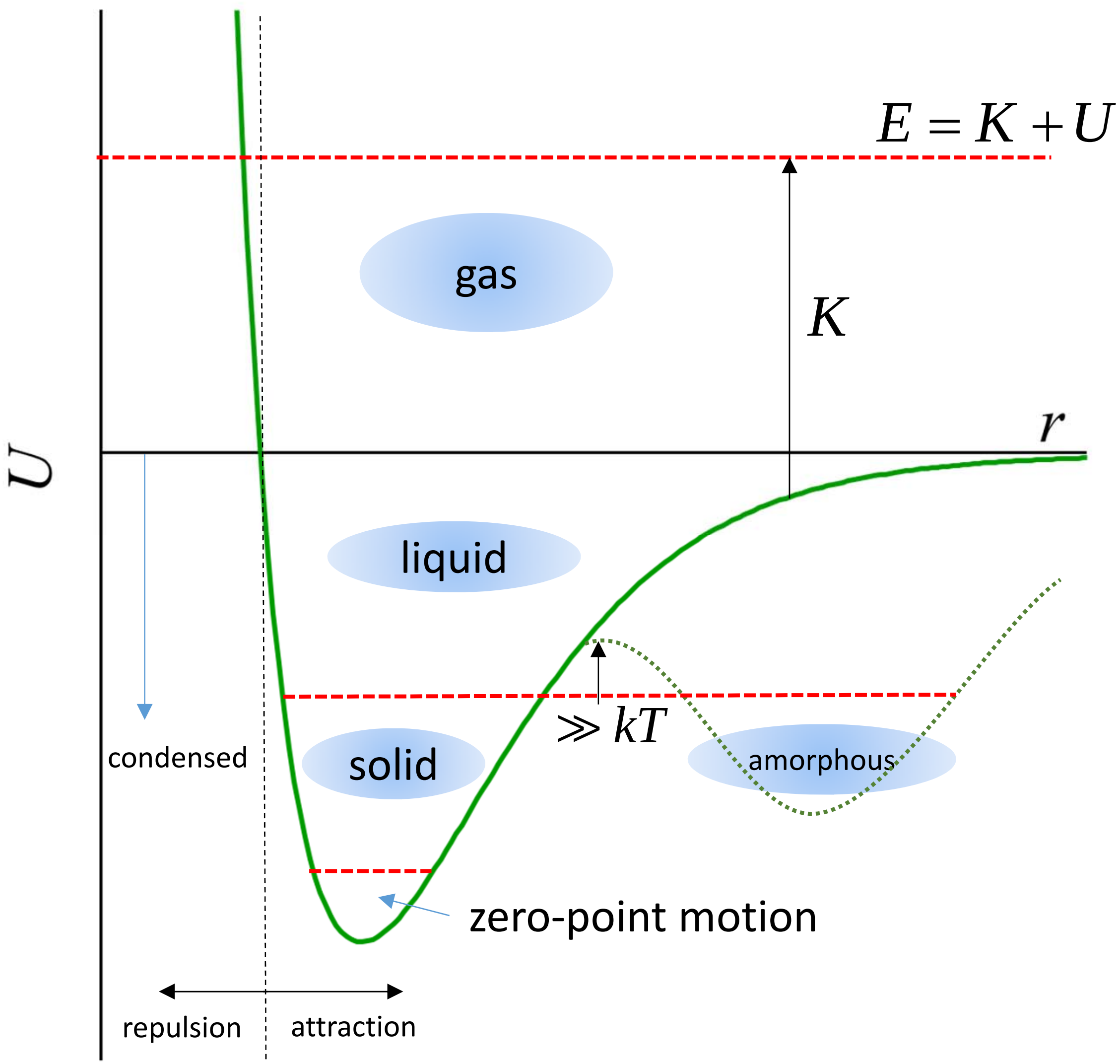
Statistical definition of entropy

Entropy



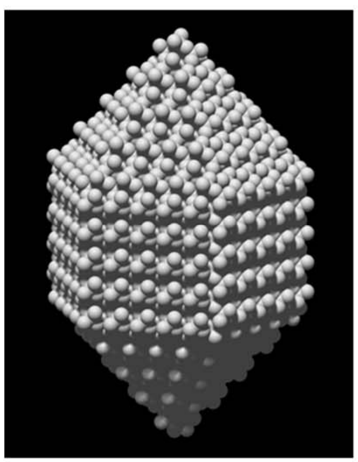
# of microstates for
a particular macrostate



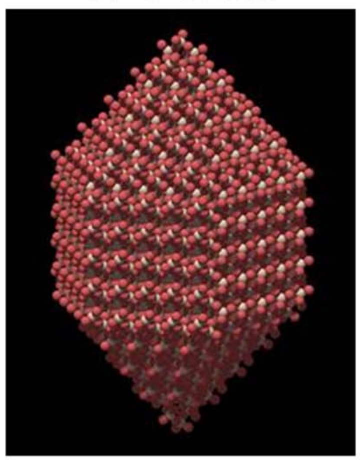
$E = K + U$
gas
K
r
U
liquid
condensed
solid
$\gg kT$
amorphous
zero-point motion
repulsion
attraction

Quartz

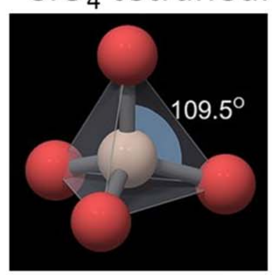
Si atoms only

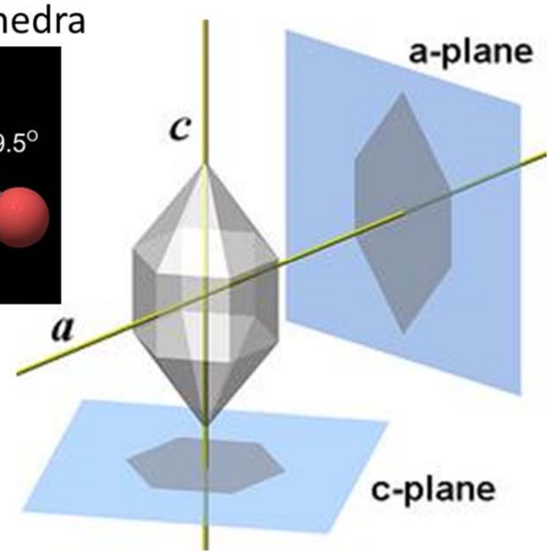


Si+O atoms



SiO_4 tetrahedra

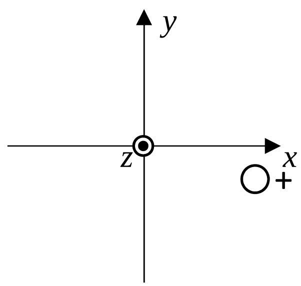




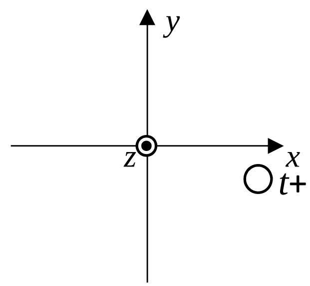
Movements

Direct Movements

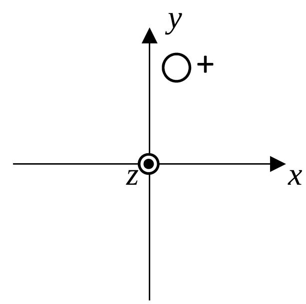
original



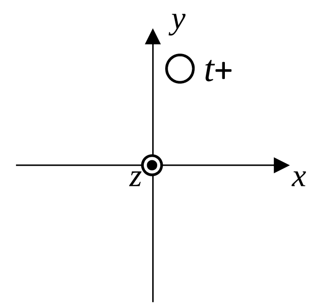
translation



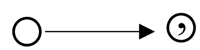
rotation



rototranslation

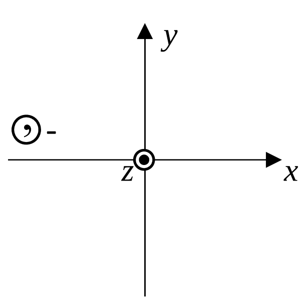


Opposite Movements

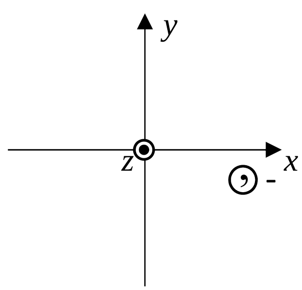


enantiomer

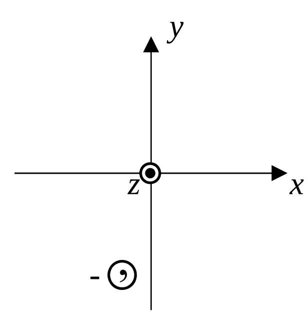
inversion



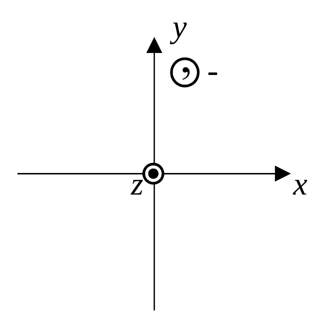
reflection



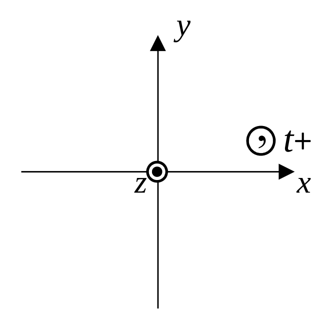
rotoinversion



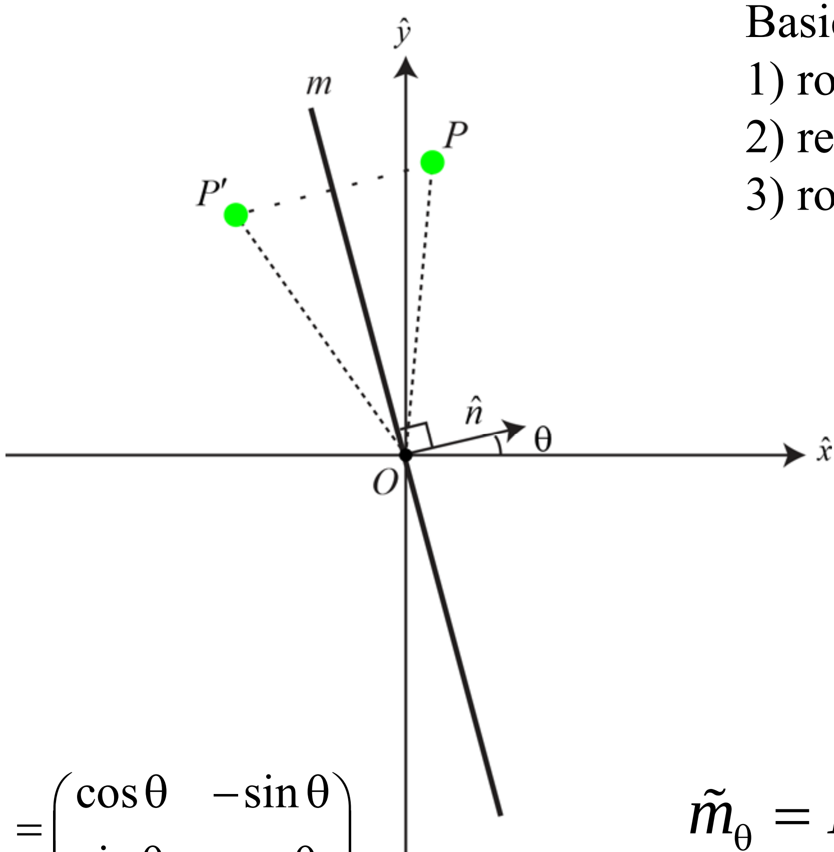
rotoreflection



glide

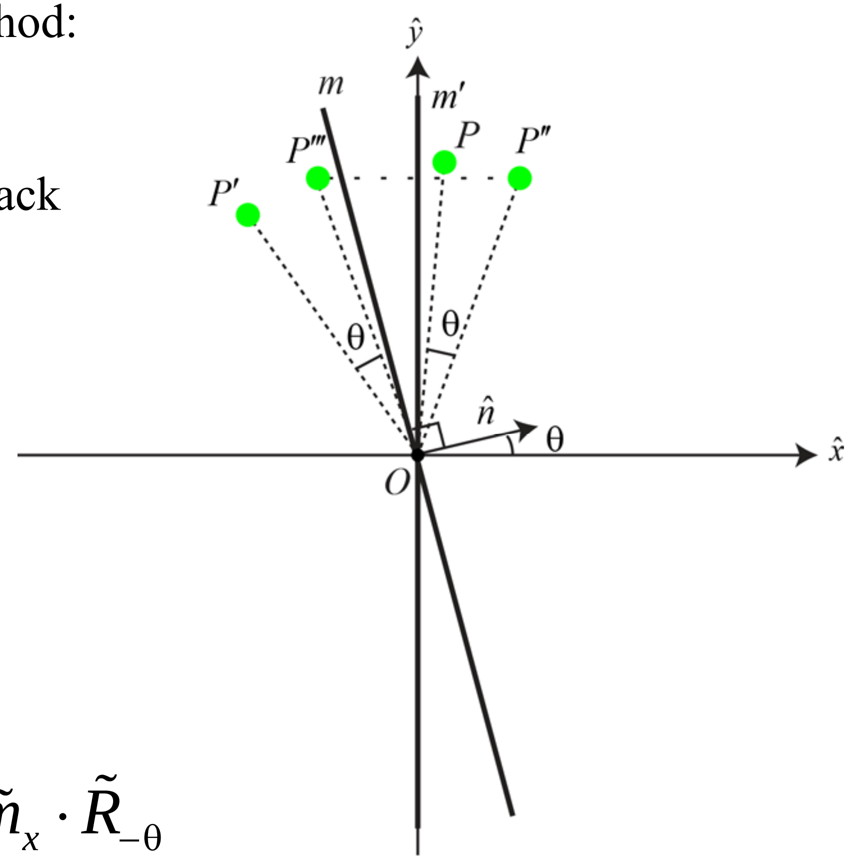


Reflection Across Rotated Line



Basic Method:

1) rotate
2) reflect
3) rotate back



$$R_\theta = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

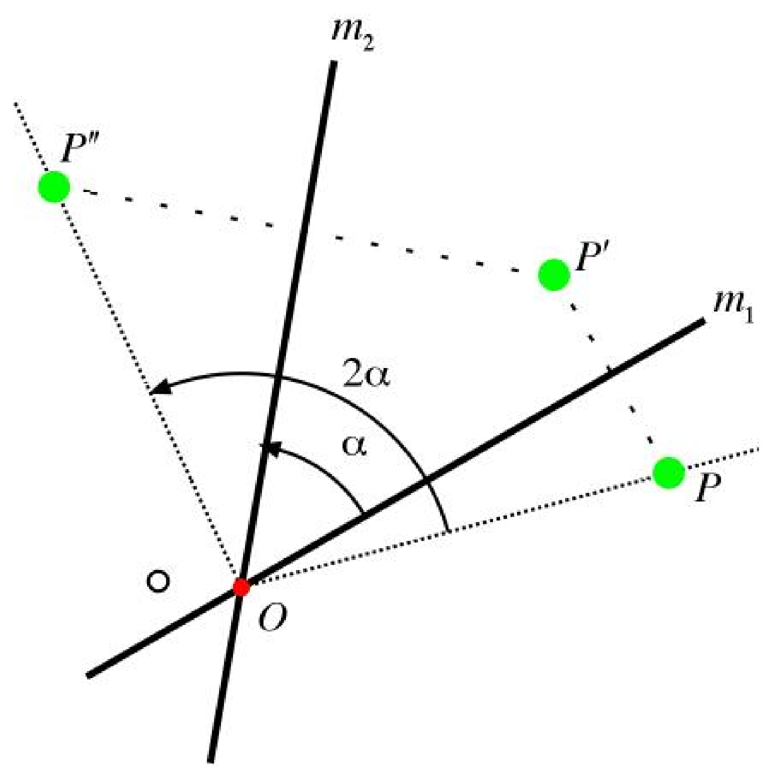
$$\tilde{m}_\theta = \tilde{R}_\theta \cdot \tilde{m}_x \cdot \tilde{R}_{-\theta}$$

$$m_x = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$m_\theta = \begin{pmatrix} -\cos(2\theta) & -\sin(2\theta) \\ -\sin(2\theta) & \cos(2\theta) \end{pmatrix}$$

$$\vec{r}' = \tilde{m}_\theta \cdot \vec{r}$$

Double Reflection

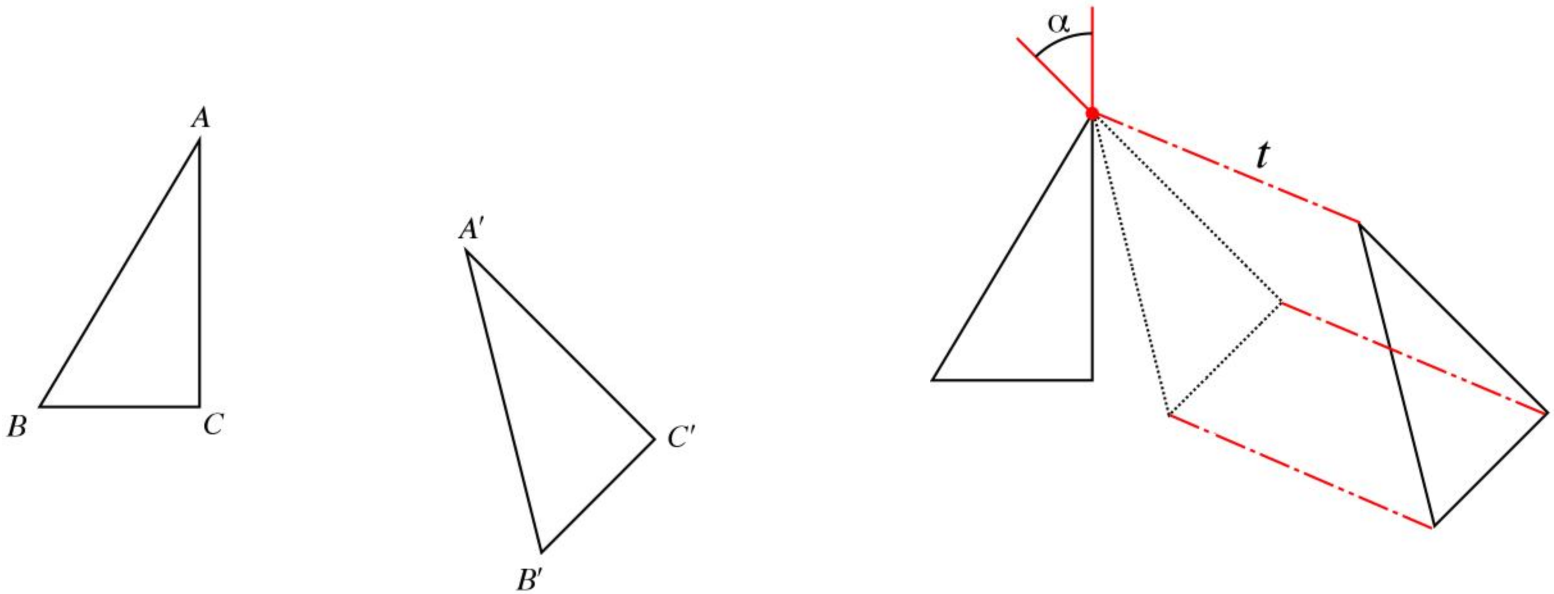


$$\mathbf{r}' = m_{\theta_2} \cdot m_{\theta_1} \cdot \mathbf{r} = \begin{pmatrix} \cos[2(\theta_2 - \theta_1)] & -\sin[2(\theta_2 - \theta_1)] \\ \sin[2(\theta_2 - \theta_1)] & \cos[2(\theta_2 - \theta_1)] \end{pmatrix} \cdot \mathbf{r} = R_{2(\theta_2 - \theta_1)} \cdot \mathbf{r}$$

Sequential reflection across two planes at an angle α
is equivalent to a rotation about their intersection by 2α.

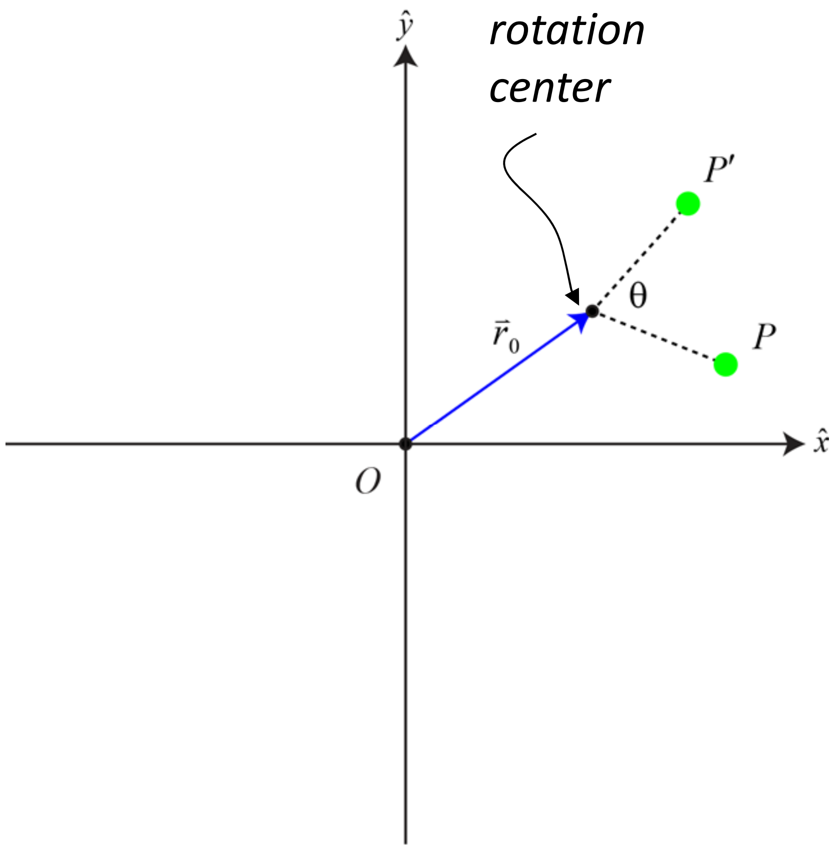
2-D Isometric Transformations

Congruent, asymmetric triangles are used as "metrics".

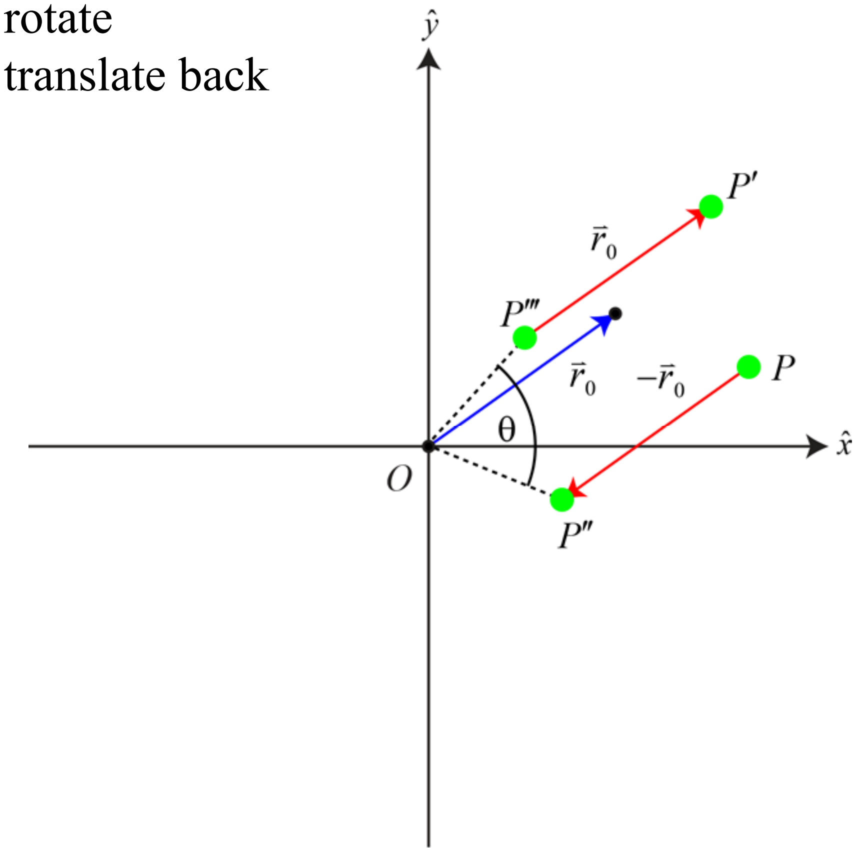


One Method: 1) Translate, 2) Rotate

Rotation About Arbitrary Point (I)



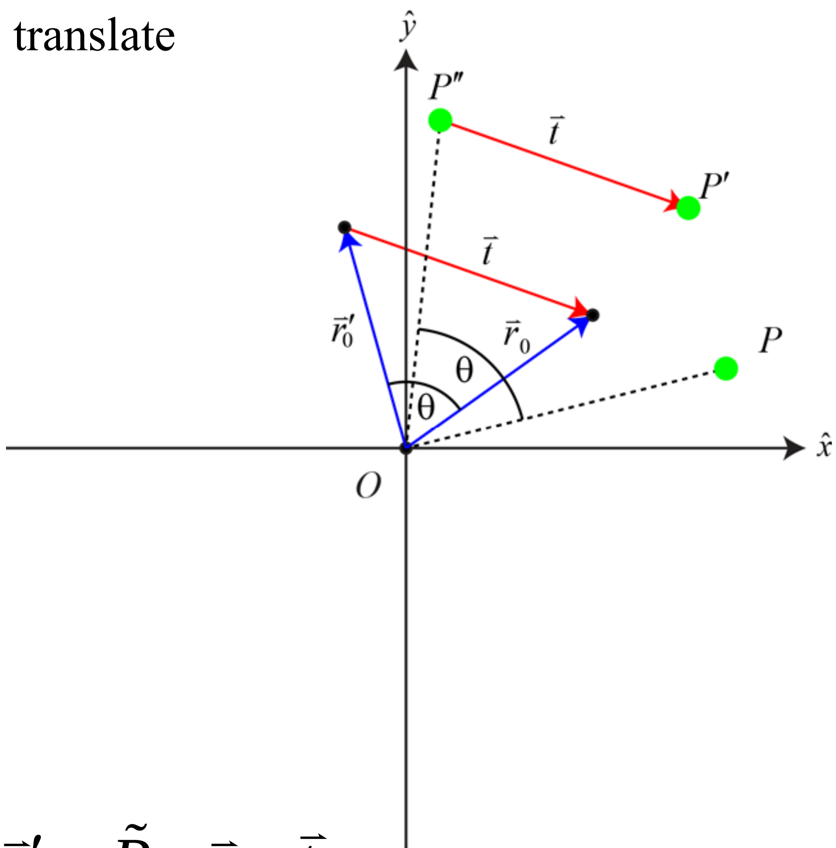
Method 1:
1) translate
2) rotate
3) translate back



$$\vec{r}\,' = \tilde{R}_\theta \cdot (\vec{r} - \vec{r}_0) + \vec{r}_0$$

Rotation About Arbitrary Point (II)

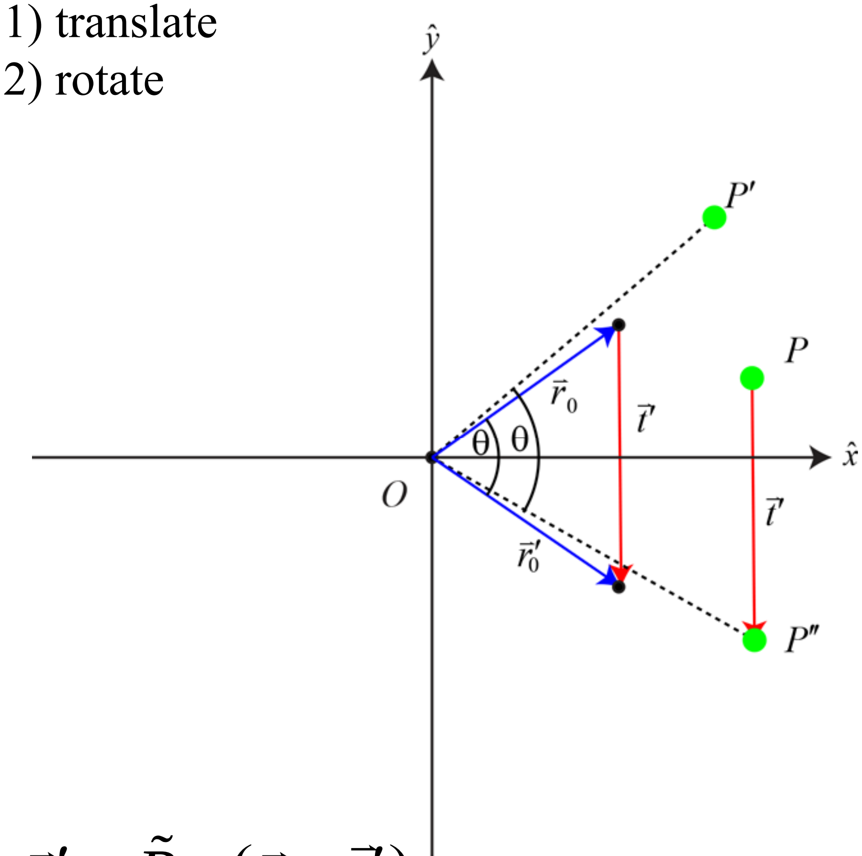
Method 2:
1) rotate
2) translate



$$\vec{r}\,' = \tilde{R}_\theta \cdot \vec{r} + \vec{t}$$

$$\vec{t} = \vec{r}_0 - \tilde{R}_\theta \cdot \vec{r}_0$$

Method 3:
1) translate
2) rotate

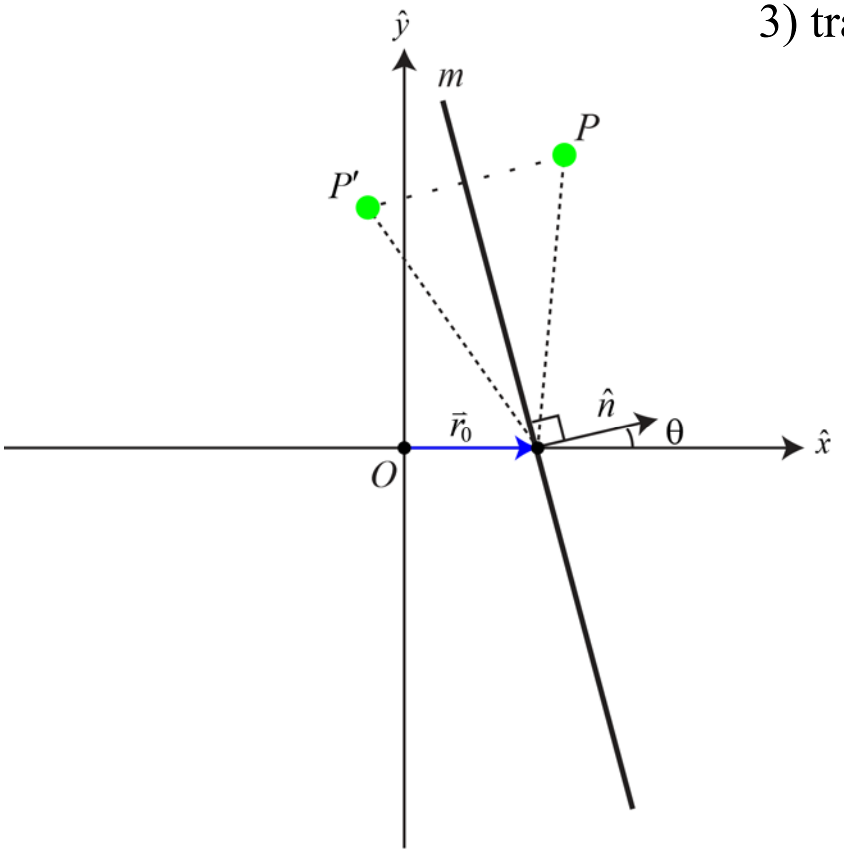


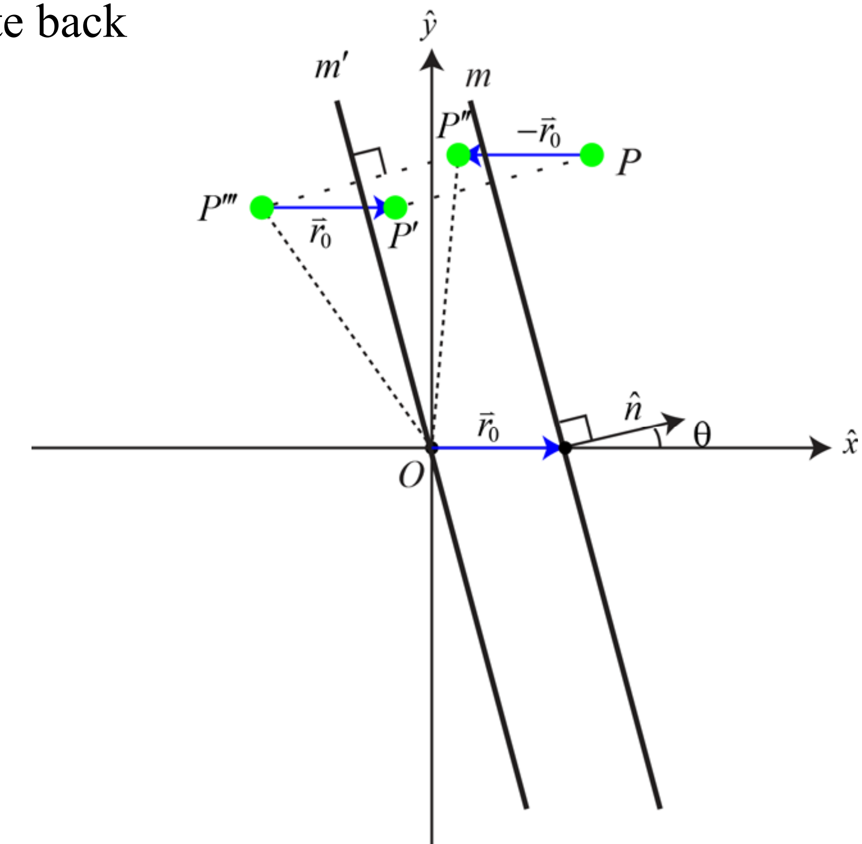
$$\vec{r}\,' = \tilde{R}_\theta \cdot (\vec{r} + \vec{t}\,')$$

$$\vec{t}\,' = \tilde{R}_{-\theta} \cdot \vec{r}_0 - \vec{r}_0$$

Reflection Across Rotated, Shifted Line (I)

Basic Method:
1) translate
2) reflect
3) translate back



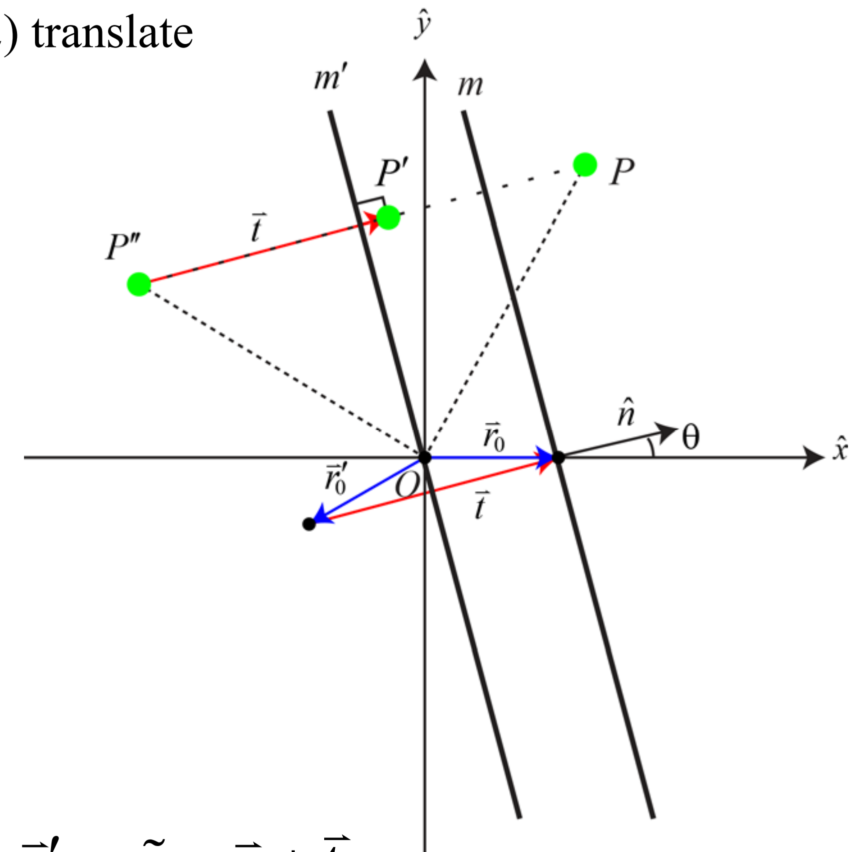


$$\vec{r}\,' = \tilde{m}_\theta \cdot (\vec{r} - \vec{r}_0) + \vec{r}_0$$

Reflection Across Rotated, Shifted Line (II)

Advanced Method 1:

1) reflect
2) translate

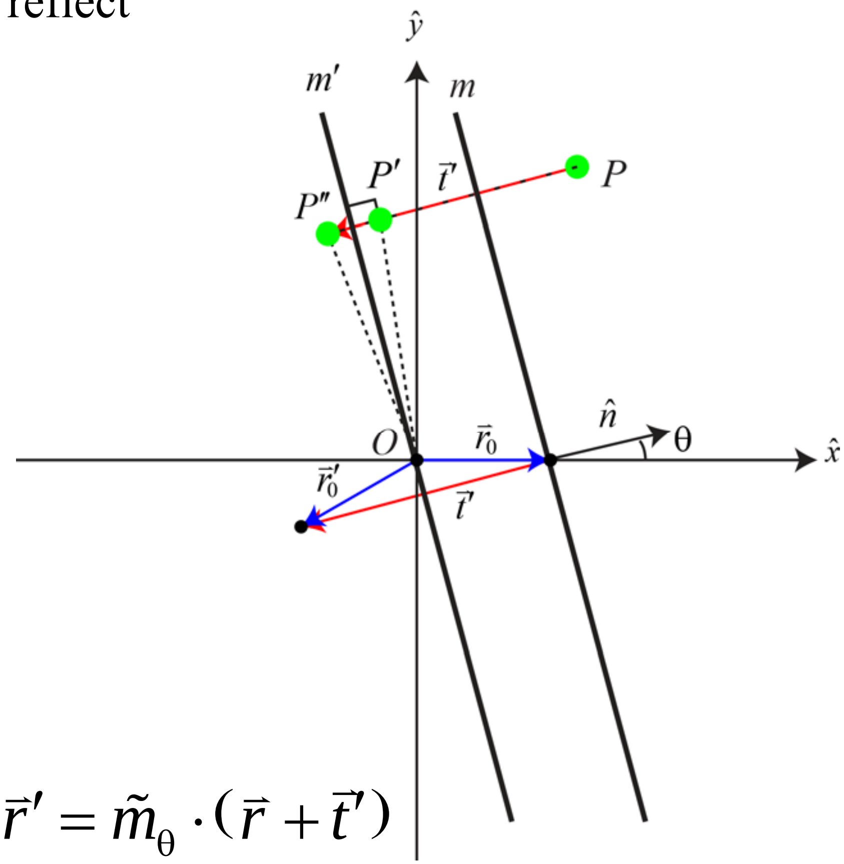


$$\vec{r}' = \tilde{m}_\theta \cdot \vec{r} + \vec{t}$$

$$\vec{t} = \vec{r}_0 - \tilde{m}_\theta \cdot \vec{r}_0$$

Advanced Method 2:

1) translate
2) reflect

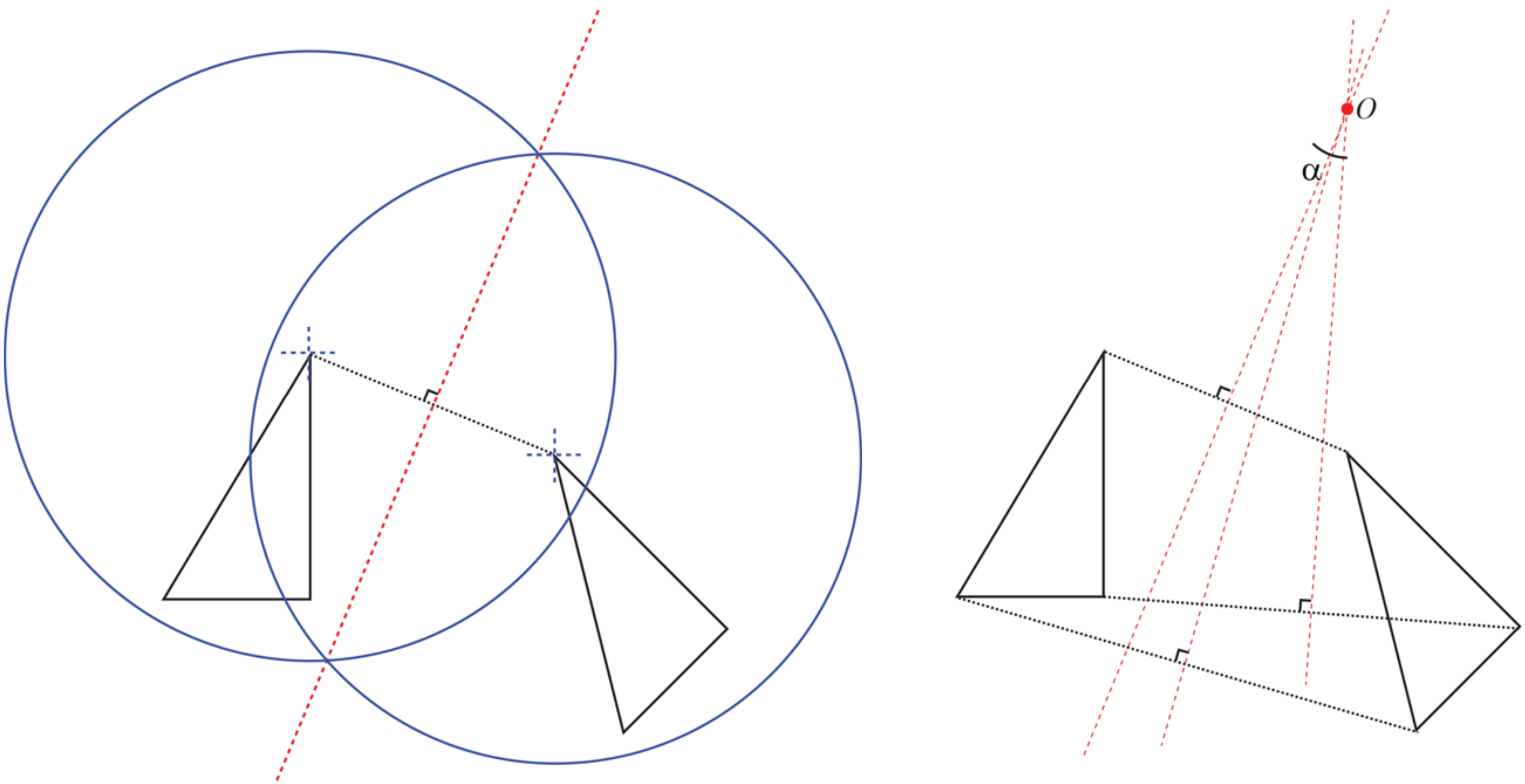


$$\vec{r}' = \tilde{m}_\theta \cdot (\vec{r} + \vec{t}')$$

$$\vec{t}' = \tilde{m}_\theta \cdot \vec{r}_0 - \vec{r}_0$$

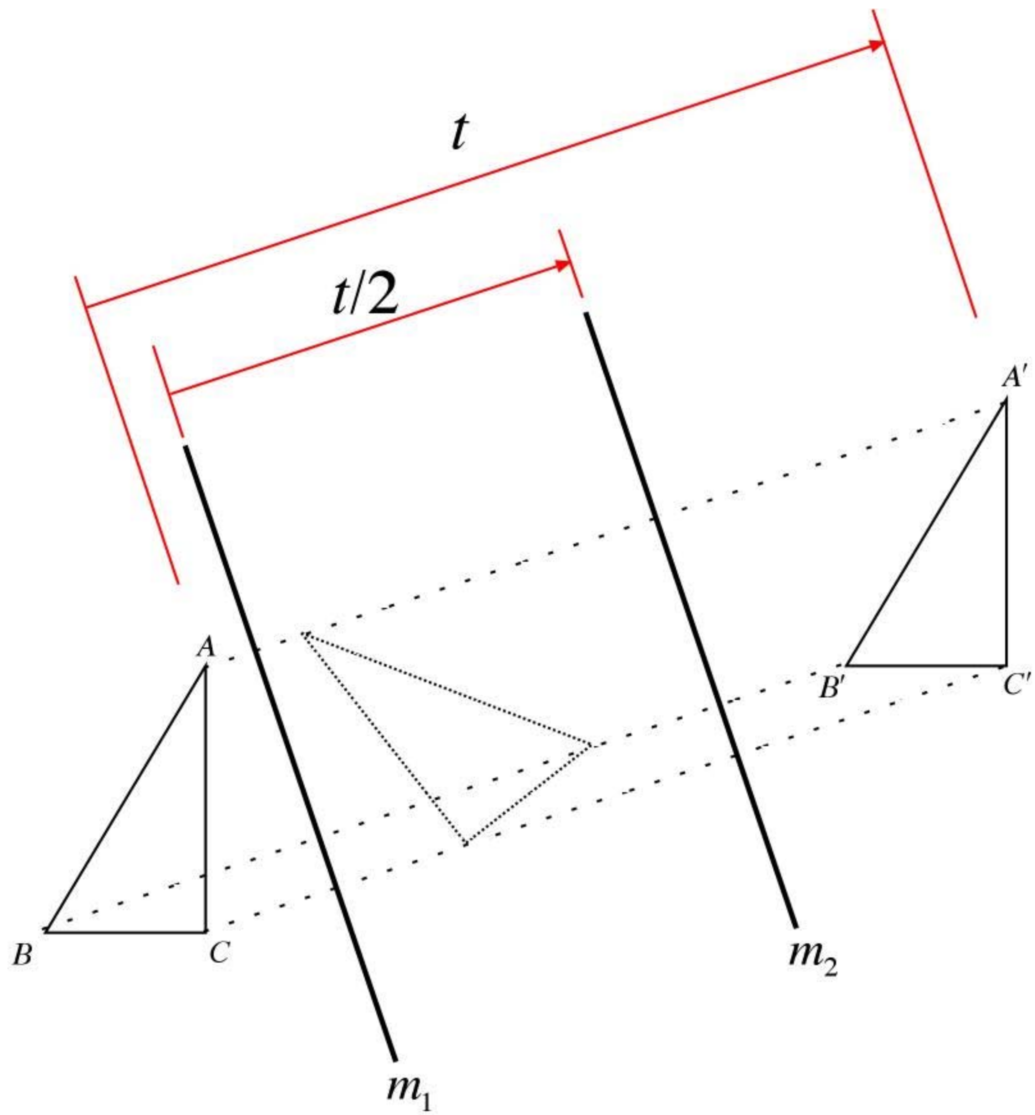
Transforming by Rotation Only

1) Find the line bisecting (perpendicular to the line connecting) each pair of points.
2) Their intersection is the rotation point, called the *Chasles center*.



Notice: Metrics look the same from *O*

Translation by Two Reflections



Equation for a line